OPTIMIZING INVENTORY-ORDERING POLICIES IN SUPPLY CHAIN MANAGEMENT: A CASE STUDY ON A SELECTED COMPANY FROM THE VAAL REGION

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DECLARATION

I, Blaise Bolan Benga Ebouele, declare that the project is my own, unaided work. It is being submitted for the requirements for Magister Technologiae: Engineering: Industrial to the department of Industrial Engineering and Operation Management at the Vaal University of Technology, Vanderbijlpark, South Africa. It has not been submitted before for any assessment to any educational institution.

B.B. Benga Ebouele

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The dissertation is the final stage of the Master degree which provides one with the opportunity to show that necessary skills and knowledge have been gained in order to organise and conduct a research project. This thesis is no exception because it has been a remarkable challenge and has introduced me to the field of Inventory Management.

I thank everyone who contributed his or her time and effort to this research project. I hereby wish to express my gratitude to the following individuals who enabled this document to be completed successfully.

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DEDICATION

This dissertation is dedicated to almighty God, my mother Benga (born Ngo Ntamack Marie-Claire), my father Benga Adolphe, my sister Benga Epossi Ernestine Elise, my niece Maka Fotso Sunshine, Papa Dacier, my second sister Nana Amelie Flora, my friend Totouom Tuekam Armand, my friend Mr Inyang Kempi, Mr Adeala adeyemi, Mr Adeyemi Stephen, and my fiancée Ntchuyou Candi Jacky Dina.

Thank you for your love and support

ABSTRACT

Implementing either periodic or continuous inventory review model within most manufacturing-companies-supply chains, as a management tool, incurs higher costs. These high costs affect the system flexibility which in turn affects the level of service required to satisfy customers. However, these effects are not clearly understood. This may be due to the fact that lead time and demand which are important input parameters of the manufacturing supply chain are not designed to be fully utilized under different and uncertain conditions such as seasonality, poor manufacturing, poor supplies and delivery performance, etc. Coming up with a hybrid inventory model which may combine, in some sense a continuous *(r, Q)* and a periodic *(R, S)* inventory review models can be useful in dealing with such problem. Therefore, more attention should be first devoted to formulating accurate models for lead time and demand that incorporate uncertainty.

This study presents a simulation based approach that assesses the effect of uncertainty on the cost of implementing a continuous *(r, Q)*, periodic *(R, S)* and hybrid inventory review models while considering appropriate constraint such as customer service and system flexibility. The stochastic representations of demand and lead time are proposed and used in the simulation models.

Results reveal that under a unique situation, implementing a continuous *(r, Q)* inventory review model may cause manager to under-budget while the use of a periodic *(R, S)* inventory review models may lead to over budget and vice versa. Further investigation shows that the cost of implementing the hybrid inventory model, although higher at the beginning of operation, seems to be the most cost effective one over time.

The result also reveal optimal re-order point path and optimal review interval path which when followed, should lead to optimal inventory cost path as demand and lead time fluctuate. Thus, a management guide is proposed that can be used by managers in making inventory decision.

***Keywords*:** stochastic Demand, stochastic Lead Time, stochastic re-order point, stochastic review-interval, inventory cost, inventory management, optimization

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GLOSSARY OF ABREVIATIONS AND DEFINITIONS

$B(t)_{con}$: Instantaneous backorder under a continuous review system

$B(t)_{hybrid}$: Instantaneous backorder under a hybrid review system

$B(t)_{per}$: Instantaneous backorder under a periodic review system

D_0 : Demand

F_i : **Probability of failure**

h : **Holding cost**

$I(t)_{con}$: **Instantaneous Inventory under a continuous review system**

$I(t)_{hybrid}$: **Instantaneous Inventory under a hybrid review system**

$I(t)_{per}$: **Instantaneous Inventory under a periodic review system**

j : **Backordering cost**

k : **Ordering cost**

$MLT(t)$: **Manufacturing lead time**

n_0 : **Number of machine**

N_{bo} : **Number of back order**

$N(t)_{con}$: **Number of replenishment at time t under a continuous review system**

$N(t)_{hybrid}$: Number of replenishment at time t under a hybrid review system

$N(t)_{per}$: Number of replenishment at time t under a periodic review system

Q : Order Quantity

t : Time

T : Period

$TC(Hybrid)$: Cost of implanting a hybrid inventory system

$TC(r,Q)$: Cost of implementing a continuous inventory (r, Q)

$TC(R,S)$: Cost of implementing a periodic inventory (R,S)

$T_{corrective}$: Reactive (corrective) maintenance time

$T_L(t)$: Manufacturing lead time under a periodic review system

T_{no} : Other time

T_{ope} : Operation time

$\mathbf{T_{pred}}$: **Predictive maintenance time**

$\mathbf{T_{prev}}$: **Preventive maintenance time**

$\mathbf{T_{su}}$: **Set up time**

U : **Utilization**

$\frac{\partial D}{\partial t}$: **Rate of demand**

ABSTRACT

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This study presents a simulation based approach that assesses the effect of uncertainty on the cost of implementing a continuous *(r, Q)*, periodic *(R, S)* and hybrid inventory review models while considering appropriate constraint such as customer service and system flexibility. The stochastic representations of demand and lead time are proposed and used in the simulation models.

Results reveal that under a unique situation, implementing a continuous *(r, Q)* inventory review model may cause manager to under-budget while the use of a periodic *(R, S)* inventory review models may lead to over budget and vice versa. Further investigation shows that the cost of implementing the hybrid inventory model, although higher at the beginning of operation, seems to be the most cost effective one over time.

The result also reveal optimal re-order point path and optimal review interval path which when followed, should lead to optimal inventory cost path as demand and lead time fluctuate. Thus, a management guide is proposed that can be used by managers in making inventory decision.

***Keywords*:** stochastic Demand, stochastic Lead Time, stochastic re-order point, stochastic review-interval, inventory cost, inventory management, optimization

Chapter 1 Introduction

According to Anderson (2008:555), inventory refers to idle goods or materials held by an organization for future use. There are many reasons why organizations should maintain inventories. The fundamental reason for doing so is that it is either practically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories customers would have to wait until their orders are filled from a source or are manufactured. In general, customers should not or cannot be allowed to wait for long periods of time. For this reason alone the carrying of inventories is necessary to almost all organizations that supply physical goods to customers (Hadley, 1963:1). However, holding inventory in the supply chain generates cost called inventory costs. In order to control those costs and act upon them effectively, a good strategy called inventory-ordering policy is therefore required. This strategy involves determining how often inventory levels must be reviewed to determine when and how much items to order (Rossetti, 2010:35). Many inventory-ordering policies have been proposed to address such issues and unfortunately none of them has effectively addressed the existing problems. These problems consist of carefully manipulating variable orders to minimize the inventory cost under a set of appropriate constraints. The present state of logistics and inventory costs is a proof that such issue still needs to be addressed.

Logistics cost in South Africa for 2009 amounted to about R323 billion (13.5% of Gross Domestic Product), while inventory cost which is one of its component was at 18.9% of this cost (Ittmann, 2011:11). From these statistics, one can conclude that supply chain partners invest significant amount of money in holding their inventories in various forms. These holding processes can cost from a minimum of 15% up to 40% of the inventory value per year (Basu, 2008:96). Most of the time, inventories occur because the timing of supply and demand does not always match (Slack, 2010:342). However, inventories are very useful in helping to meet customer demand, avoid loss of sale and to meet unexpected increase in demand. Although statistics show that inventory costs are not the greatest component in logistics cost, they represent a considerable amount of money tied up in investments that are not producing any return and, in fact, incurring cost (Mangang, 2008:90). Therefore, finding ways to appropriately reduce them is the key concern of inventory managers.

South African manufacturing companies are facing the problem of meeting the required inventory levels for some products in their supply chains. This is typical to such entities since, according to Simchi-Levi (2008:31), managing inventory in complex supply chains is typically difficult and inventory-related cost decisions can have a significant impact on the customer service level and supply chain system cost. As a result, the recurring question is how production and distribution should be scheduled so as to minimize inventory cost while keeping customers satisfied and also maintaining supply chain flexibility. The answer to this question still remains elusive because the demand for the firm's product and the protection demand interval of inventories may vary widely. Although the use of alternative inventory review policies has been seen as one part of the solution, identifying the most cost effective and flexible one is a current challenge.

1.6 Background

Periodic *(R, S)* and continuous *(r, Q)* inventory review system are the two classical inventory review models or system that have been commonly used in the supply chain to deal with inventory problems. In continuous review system *(r, Q)* the level of inventory is assumed to be checked continuously so that an order ***Q*** can be placed whenever the re-order point ***r*** is reached (Anderson, 2008:584). However, in periodic review system *(R, S)* the level of inventory is checked after some given and known time interval ***R*** and enough items are ordered to raise the inventory position to the maximum level ***S*** (Chiang & Rossetti, 2010:352). Both inventory review models have advantages and disadvantages. That is why one may think of an approach that combines, in some sense, the feature of both periodic *(R, S)* and continuous *(r, Q)* inventory review models in order to come up with a hybrid inventory model.

Comparing periodic *(R, S)*, continuous *(r, Q)* and hybrid review systems in order to identify the one that is able to adapt to a stochastic supply chain at all time is the key concern of this research project. Thus, the parameters which impact the total inventory cost are determined and analysed.

Previous studies used a simulation approach to analyse alternatives inventories review policies in the supply chain. Unfortunately, these studies did not effectively address the issue of the optimal cost associated with implementing either a flexible periodic *(R, S)* or a continuous *(r, Q)* or a hybrid inventory review policy. For instance, Xudong, (2008:18) developed a discrete event simulation-based approach to evaluate the impact of a periodic *(R, S)* and continuous *(r, Q)* inventory review policies on performance measures such as cost and customer service level. According to Setamanit (2011:1), the study by Xudong, (2008:18) explored the situation where inventories in the supply chain were assumed to have parallel paths with different ordering policies. These configurations analysed by Xudong produced different results. Merkuryeva (2008:434) undertook a simulation-based approach in order to investigate the gap between cost of periodic review model *(R, S)* and continuous review model *(r, Q)* in condition of demand variability and uncertainty. The coefficient of demand variation, capacity utilization and number of echelon were chosen as parameters that affect the supply chain. The comparative study between the two mentioned inventory policies was based on the analysis of the costs gap and the Additional Cost of Cyclic Solution (ACCS). After analysis, the cost difference between the two inventory review policies was not clearly established. Recently, Setamanit (2011:4) focused on the issue related to determining the optimal review system that lead to higher supply chain performances. Lead time and customer demand were chosen as the inputs that affect the supply chain system. The comparison was made on a minimized inventory cost subject to a specified service level. It was found that the average total cost from both policies was not statistically different. It led to the conclusion that the implementation of either a periodic or continuous inventory review model in a supply chain would never produce significant cost difference. This result can be challenged. That is the reason why another approach is proposed in this study. Hence, an important issue is "how to use an approach that will be different from the work of Setamanit (2011:4)".

The difference between the two approaches may lie in the nature of variation in demand and lead time. Unlike the work of Setamanit (2011:4), where the steady increase of demand and lead time are dealt with, this dissertation deals with variation of demand and lead time with fluctuation. i.e. increase in demand with fluctuation and fluctuating lead time (see figure 7). It is quite clear that there is a gap to fill here since the stochastic nature of the supply chain and many other factors that characterize a real supply chain have never been fully addressed.

1.7 Rationale and motivation

Supply chain management should be an integrated approach to the planning and control of inventory, throughout the entire network in order to satisfy end customer's demands (Thangam, 2009:137). Even though periodic *(R, S)* and continuous *(r, Q)* inventory review policies are the most commonly used inventory management tools in the supply chain, the cost difference between them has never been clearly established. This may be alluded to the fact that the models for those different types of review policies are not clearly understood and as such well-established. Thus, determining the parameters that can be appropriately employed to maximize the benefits of the two mentioned inventory policies is still crucial.

The existing literature on inventory management and supply chain has not effectively established the difference between the different types of inventory policies as justified below: Firstly, the relationship that exists between necessary parameters of diverse inventory models and the total cost of inventories should be analysed according to new techniques (Altman, 1983:307). Unfortunately, these techniques have never been effectively employed so far (Xudong, & Merkuryeva, & Setamanit, 2010:1). For instance, in continuous review policy *(r, Q)* the level of inventory is assumed to be checked continuously so that an order ***Q*** can be placed whenever the re-order point ***r*** is reached (Anderson, 2008:584). The quantity ***r*** is set to protect against the possibility of shortage of the product during the lead time ***L***. Contrary to continuous review policy, in the periodic review policy, the inventory level must be checked after some given and known time interval ***R*** and enough items are ordered to raise the inventory position to the level ***S*** (Chiang & Rossetti, 2010:352). The level **S** is designed to cover the demand for the product over the lead time plus the length of the review period ***(L+R)***. In this case, ***(L)*** and ***(L+R)*** are referred to as the protection demand intervals under a continuous *(r,Q)* and periodic *(R,S)* inventory review policies respectively. They are the time interval that a firm must rely on its safety stocks to protect against a stock out (Drake, 2008:123). The costs of operating both review policies in uncertain supply chains could be associated to these parameters (e.g protection demand intervals and order quantity). One good reason is that within a real supply chain, the input variables (lead time and demand) and decision variables (protection demand interval and order quantity of inventories) may change over time due to many reasons such as seasonality, poor manufacturing performance, poor supplier performance, poor delivery performance (Patil, 2012:304). Unfortunately, the current parameters of these models are not designed to be fully utilized under different and uncertain circumstances. An immediate consequence, for example, is that the applications of periodic review model *(R, S),* may create a stochastic review interval (Waller, 2008:245). Failure to effectively define and analyse the protection demand intervals might reduce firm’s ability to order the appropriate amount of inventories required to satisfy customers at all time and deal with random fluctuations in demand and delivery time. This might lead to exorbitant inventory costs. Seen from this angle, it is clear that a stochastic protection demand interval and quantity of items ordered may have a significant impact on the total inventory cost, if not managed properly. That is why the existing relationship between the mentioned variables should be further investigated and analysed. It would make the analysis more realistic.

Another flaw of previous researches is connected to the use of fewer set of performance measures required to effectively compare both inventory policies in a real supply chain. Previous researches focused solely on the comparison of alternative inventory ordering policies upon two types of performance measures, which are the high level of efficiency and services (Merkuryeva & Setamanit, 2011:1). There is an important factor missing in this approach, which is flexibility (Beaumont, 1999:273). Although a supply chain may be operating under minimum cost and a good service level, it may demonstrate a lack of flexibility to meet random fluctuation in demand and delivery time. Flexibility measures need to be introduced to estimate supply chain cost (Wu, 2008:2222). As result, comparing a periodic *(R, S)* and a continuous *(r, q)* inventory models upon performance measures including the high level of efficiency, service and the ability to respond effectively to a changing environment is one approach worth spending time on. These three performances are all important because they measure the most pertinent aspects of every supply chain (Beaumont, 1999:273).

The final flaw observed on previous researches is linked to the use of inappropriate inventory models. Classical inventory models, (for periodic *(R, S)* and continuous *(r, Q)* inventory models), most of the time, are used independently to study a system and to predict the effect of change to that system with respect to certain conditions. However, this approach is not robust enough to adapt to a changing environment. That is why using a tool such a hybrid model, which is a combination of a periodic *(R, S)* and a continuous *(r, Q)* inventory models, will help to deeply understand the inventory cost. Currently, inventory control problems in this area are solved by using deterministic simulation approach (Pawlack, 2007:12). Unfortunately, the stochastic nature and complexity of supply chain systems are usually considered as an obstacle (Harkan, 2007:330). Consequently, modelling and simulating the stochastic supply chain system seem to be a novel for supply chain performances analysis (Jianghong, 2010:1).

It is quite clear to believe that an effective comparison approach has to be performed. Hence, another most important issue is "how to evaluate the impact of using periodic *(R, S)*, continuous *(r, Q)*, and hybrid inventory review models on appropriate supply chain performance measures that may include the annual total cost, stock out probability and system's flexibility". The first approach to address this issue is to figure out how multiple performance measures and data variables can be used for a comparison study within an uncertain supply chain. The second is to use an appropriate modelling and simulation optimization approach.

1.8 Problem Statement

Implementing either periodic *(R, S)* or continuous *(r, Q)* inventory review model within most manufacturing-companies-supply chains as a management tool may incur higher costs (Setamanit et al, 2010:1). These high costs affect the system flexibility which in turn affects the level of service required to satisfy customers. However, these effects are not clearly understood because the parameters of both inventory review policies (protection demand interval, order quantity, etc) are not designed to be fully utilized under different and uncertain conditions such as seasonality, poor manufacturing, poor supplies and delivery performance. Coming up with a hybrid model which may combine, in some sense, the feature of both continuous *(r, Q)* and a periodic *(R, S)* inventory review models should be useful. Therefore, there is a need to evaluate the impact of using the three different models on a set of appropriate performance measures that may include the annual total cost, stock out probability and system's flexibility in order to search for the most cost effective inventory review model. This work also seek to find the optimal sets of parameters of inventory management under stochastic condition so as to optimise each policy independently.

1.9 Purpose of the Study

The purpose of this study is to evaluate the impact of using a flexible periodic *(R, S)*, continuous *(r, Q)* and hybrid inventory review policies on appropriate set of supply chain performance measures.

1.10 Objective and specific objectives

The main objective of this research project has been to determine the most cost effective inventory review model on a set of supply chain performance measures that included annual total cost of inventories, stock out probability and system's flexibility. The specific objectives required to accomplish the main objective were to:

- Assess the supply chain annual total cost, stock out probability and system flexibility on implementing the three inventory models: periodic *(R, S)*, continuous *(r, Q)* and hybrid inventory models
- Establish flexible inventory review models
- Measure the effect of changes in input parameters on the three inventory model's performances.
- Determine the main constraints

  A feasible region that defines limits of stock out probability and system flexibility of the supply chain (Restriction that limit the possible solution to the problem)
- Measure the objective function

  The value of protection demand interval and re-order quantity that minimize the annual total cost while satisfying the constraint of the stock out probability and system flexibility
- Establish the relationship between changing protection demand interval, re-order quantity and the annual total cost change of inventories (for the three inventory models)
- Establish the relationship between changing cost of periodic *(R, S)*, continuous *(r, Q)* and the hybrid inventory model
- Compare the above and select the best inventory model to be employed under given set of conditions.

1.6 Contribution of the research

The current study is aimed at addressing or contributing solutions to these issues:

- Help companies (ie their inventory managers) to better understand and implement the right inventory model based on the situation at hand
- Help inventory managers to reduce Cost
- Help firms to maintain and strive for strategic position in the market
- Allow Vaal University of Technology to benefit on peer-review research outputs (See appendix 1)

1.7 Overview of the dissertation

This thesis reports on the determination of the most cost effective inventory review model on a set of supply chain performance measures that may include annual total cost of inventories, stock out probability and system's flexibility. The introduction of the problem, purpose, goal and specific objectives are presented in this first chapter. The next chapters are organized in the following order.

Chapter two summarizes the role of inventories and some problems encountered in the context of supply chain management. In the first part of the chapter, basic terms and concepts related to supply chain management are defined. In the second part of the chapter, inventories are defined and discussed as the object of supply chain in more detail. Other topics discussed include the type of inventories, the purpose of inventories, type of inventory review policies and cost related to these inventory review policies. Then, in the third part of this chapter, some important parameters such as customer's demand and lead time are presented and issues related to them are discussed. Finally, the cost model of implementing continuous (r, Q) and periodic (R, S) inventory review policies are described and issues related to them are discussed as well.

Chapter three proposes on how to find solution of the inventory problem. The first part of this chapter presents step by step approach or information on how a flexible continuous (r, Q) inventory review model can be developed and formulated. In the second part of this operation, the same step by step approach is undertook for a periodic (R, S) inventory review model. The last part of this chapter shows how one may hybridise the two inventory models.

Chapter four presents the results obtained from a simulation study performed in order to test whether the solution proposed to the mentioned problem work well or not.

Finally, the dissertation is concluded with the outcome of this research in chapter five. Recommendations are also provided to assist the manufacturing company improve their approach to inventory management.

Chapter 2 Literature and theoretical background

2.1 Introduction

This section focuses on the literature and theory that provide the foundation to the research study and to which the possible finding will provide a contribution. Note that the scientific literature on comparing inventory review systems is huge and it is not easy to provide a comprehensive overview of all literature. Therefore, this literature section focuses on those recent publications directly related to the problem being addressed. The following three areas of literature are reviewed in this section.

- ❖ Basic term and concept related to supply chain and inventory management
- ❖ Theory related to inventory level, inventory cost, lead time and demand

    - Demand
    - Lead time
    - The role of inventory
    - Type of inventory
    - Reason for holding inventory
    - Decision to make regarding the creation and holding of inventory
    - The relationship between inventory level, consumption rate and lead time
    - The inventory cost model

- Periodic *(R, S)*, continuous *(r, Q)* and a hybrid inventory cost models and issues

2.2 Basic terms and concepts related to supply chain and inventory management

According to Oxford Dictionary (2008:1486), supply chain is defined as the series of processes involved in the production and supply of goods: from when they are first made, grown, etc. until they are bought or used. In other word, a supply chain is composed of a sequence of organizations, beginning with the basic suppliers of raw materials, and extends all the way up to the final customers (Peter W G Morris, 2007: 226). Recent literature defines supply chain management as the integration of supply and demand processes, both within and across companies, such that supply chain performance may be improved by the incorporation of market intelligence, including consumer demand information, into supply decision making (Eroglu, 2011:1). Supply chain management can also be seen as an integrated approach to the planning and control of inventory, throughout the entire network in order to satisfy end customer's demand (Thangam, 2009:137).

Anderson, (2008:555), said that inventory refers to idle goods or materials held by an organization for use sometime in the future. Items carried in inventory include raw materials, purchased parts, components, sub-assemblies, work-in-process, finished goods, and supply.

Stock out (shortage) is a term used to refer to a situation where no stock or inventory is available to fulfil a request from a customer or production order during a peack operation. Stock out can be costly, including the profit lost for not having the item available for sale, lost goodwill and substitutions (Vitasek, 2006:136). In the same order of idea, safety stock can be defined as the inventory that is held to prevent stock-out.

Backorders are usually caused by stock out. As such, back order inventory can be defined as a customer order that cannot be fulfilled and which the customer is prepared to wait for some time (Vitasek, 2006:14). The number of backorders per period are important measures of the quality of company's customer service and the effectiveness of its inventory management.

All those activities combined to ensure that the proper stock levels are maintained, at all times, is defined as inventory management (Ostrowet al & Simchi-Levi & Kelton & Banks, 2009:214).

Flexible inventory system is an inventory system in which there is some amount of flexibility that allows the system to react in case of changes, whether predicted or unpredicted. Such system has the ability to react and adapt to random fluctuation in demand and lead time.

2.3 Theory of demand, lead time, inventory level and inventory cost

2.3.1 Demand

Demand for a product can be defined as the quantity of product a consumer will purchase. There are many factors that affect demand variability. According to Fogarty et al, (1991:51-53) these fluctuations can be attributed to external factors such as rapid changes in consumer preferences or events affecting geographical region such as major earthquakes or natural disasters, major sport games. Occasionally, fluctuation may also be due to marketing efforts which has successfully piqued the consumer's interest in the product. The supply structure in the economy can also affect the nature. For all these reasons, the appropriate nature of demand for a product has to be forecasted.

Demand estimation is one of the most important aspect of managerial economics, since a firm would not be established or survive if sufficient demand for its product do not exist or could not be created. If a manager is successful in determining the appropriate demand function based on important variables, then the decision making process by the manager may be valid for the business (Raza, 2013:27). The best estimated demand function may enable the management to forecast demand with few errors.

Hornby (2005:582) states that demand forecasting is the activity of estimating the quantity of product or service that consumers will purchase based on available historical data. In a competitive market environment, it is very difficult to forecast demand. Although many probabilistic model have been proposed using various demand pattern, most of the literature dealing with probabilistic inventory models assumes the probability distribution of the demand rate (Sarbjit & Shah, 2010:5).

There are many other papers which talk about various techniques in managing the level of uncertainty (Lee & Papachristos, 2009: 126). While some of the papers are focused on the problem of attributing a specific probability distribution in inventory control policies and demand, they are still describing the issue for given or fixed context (Setamanit, 2011:1). They emphasize that the random effect that causes demand to deviate from its average level may be estimated from distribution which has a mean that can be predicted and standard deviation that can only be estimated from past data (Setamanit, 2011:2).

It is well known that one area of focus of inventory managers is the ability to use accurate model for demand. In reality, inventory managers want to know if it is reasonable to assume either a constant, increasing, decreasing or seasonal demand with fluctuation. The answer to this question may depend on the type of product, and the market, technology used to build the product. Time series method can effectively be used to forecast demand. The goal is to isolate pattern in past demand data such as trend, seasonality, cycle, randomness (see figure 1).

From observing figure 1, one can infer the existence of a specific pattern in the data collected. This shows that a model for demand can be expressed in the form of increasing line with fluctuation as follow (Shah, 2009:172).

$$\begin{aligned} D(t) &= Systematic_{Part} + \varepsilon(t) \\ D(t) &= ((Level_{term} + Trend_{parameter} * t) * Seasonalit_{parameter}(t)) + \varepsilon(t) \end{aligned} \qquad (1)$$

Where the *level term* (estimate of demand) indicates the short term pattern of demand that are not repetitive or the estimated value of demand at the initial time, the trend term really indicates demand rate (growth rate or decline rate or both), the seasonal pattern describes demand when it is influenced by seasonal factors (weather, festival, organization policy), *t* is the forecasted period, *ε(t)* is the random part which capture the effect of all the unpredictable factors that cannot reasonably be included in the model as explanatory variable.

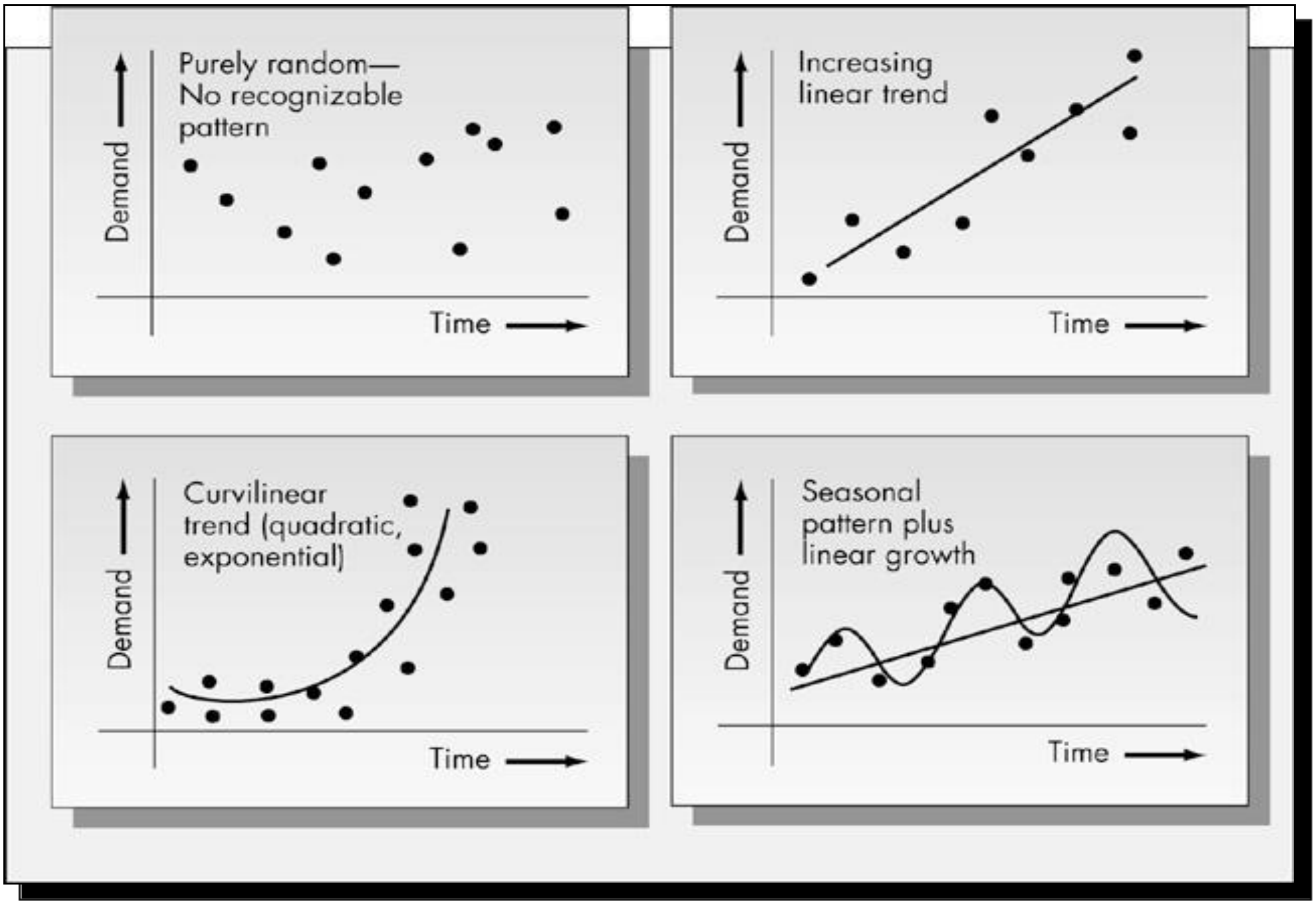


Figure 1 Type of demand pattern (Makridakis et al, 1989:601)

As seen in equation (1), the systematic part can be decomposed into three forms: Level, trend and seasonality. A seasonal pattern exists when demand is influenced by seasonal factors. Such seasonality pattern can be easily identified if data are plotted for several periods. The same principle can be applied to all form of demand.

It is also known that uncertainty in demand of product can be reduced by forecasting methods. There are various forecasting models for demand that can be used in the analysis. These forecasting model includes simple moving average method, single exponential method, and double exponential method, triple exponential method, linear regression method, multiple regression method, semi average method, and naïve model (Heizer & Sharma, 2013:137). It is also known that selecting the best forecasting tool to be used is not easy (Neale, 2009:388). Forecasting demand is an unavoidable operation. Thus, the most appropriate forecasting method is required if one want to improve its inventory system cost.

For example, (Raza & Gujarati, 2013:27) used regression analysis to estimate demand. In using regression analysis, the demand estimation can start by examining the determinants of demand for a commodity. There may be several determinants of demand function such as price of commodity, price of substitute commodity, price of complementary product, income and preferences of the consumer, quality of the product and advertisement etc.

It should be noticed that any approach that can enhance the forecasting process should be well accepted. Therefore, multiple regression can also be used to estimate the nature of the relationship between a dependent variable and several independent variables. According to Shah (2009:171), when there is only one dependent variable and one independent variable involved in the relationship, a simple regression model should be used. Because the lead time is always dependent of demand, it is also necessary to study those parameters conjointly. Therefore, regression analysis can be applied to demand, lead time or both. The next section deals with the lead time and its issues in more detail so that one can clearly see its importance.

2.3.2 Lead time

Lead time, which is one of the important parameter is defined as the time that elapses between the placement of an order and the receipt of the order (Senapati, 2012:105). It is usually defined for either a stock-pile-up inventory or stock-shipment inventory. In most stochastic inventory models encountered in the literature, the optimal decision variable are determined with the assumption that lead time is a fixed or constant parameter (Harkan, 2007:328). But lead time is composed of many controllable components such as run time, set-up time, waiting time, moving time and lot size inspection time (Harkan & Groover, 2007:328). However, the lead time may influence customer service satisfaction and impact inventory costs. These effects of lead time are well known but are too general to be used in practical ways. In fact, under practical situation, lead time should be reduced. Consequently, it is important to know how and to what extent each of the many components of the lead time influence the level of inventory in order to select the most cost effective inventory model.

There is a growing literature on modelling the effects of changing lead times on inventory control models. Many studies have dealt with lead time reduction in the supply chain. (Hsiao et al, 2008:3). Further, the effects of reducing the lead time on the backorder rate under a periodic review inventory was studied (Lin, 2006:125). Results that came from these studies according to Lin (2006, 125), although constructive, can be improved. Later, many methods for reducing lead times and their impacts on the safety stock and the expected total cost of a continuous *(r, Q)* inventory review models were studied (Glock, 2012: 37). Inventory models with stochastic demand and variable lot size-dependent lead time were analysed. It was assumed that lead time consists of production, set-up and transportation times. As a consequence, it was found that lead time may be reduced by increasing producing rate or by reducing the lot size (Glock, 2012: 38). In many practical ways, by reducing the lead time, it is possible to lower the safety stock, reduce the stock-out level and improve the customer service level. It should be noted that this present dissertation seeks to improve on previous results by considering random downtimes (or maintenance) and random demands (or stochastic lot size). It is clear that the inventory level at any time depends on the actual inventory policy implemented and its environment.

In classical periodic *(R, S)* and continuous *(r, Q)* inventory review models with deterministic demand and lead time, the problem of shortage is easy to address. It is possible to predict what the inventory level would be when an order arrives. This is not easy when the demand and lead time are random (Chiang, 2008:433). For instance, there are situations where the replenishment may take even longer. This situation may happen if there are disruptions at the producer level. That is why it is necessary to carry some additional stock to avoid stock-out costs that may occur, especially when the lead time is also random. This stock, known as safety stock, is defined as the expected value of the net inventory at the time an order arrives. In reality, when the fluctuations in demand are high, a reasonable amount of safety stocks are required to avoid stock-out, and as a result, holding costs are increased. This issue indicates that controlling the lead time should be one of the principal concern under a backordered case because it directly affects the safety stock level. Unfortunately, for classical periodic *(R, S)* and continuous *(r, Q)* inventory review models, the lead time and safety stock are rarely designed to be fully utilized or to fit in the environment in which they are used. From reading the literature related to demand and lead time, one can clearly see that new model for these important input parameters are therefore necessary because they can help to determine the appropriate inventory level.

2.3.3 The role of inventory in the context of supply chain

All organizations keep inventories. Inventories exist in the supply chain because the timing of supply and demand does not always match. According to Closs et al (2002:12), inventory plays an important role in four aspects of all businesses. First of all, inventory enables the business to achieve economies of scale. It allows economy of scale within a single facility and permits each process to operate at maximum efficiency and utilisation rather than being constrained by limited resource. Secondly, inventory balances demand and supply within the firm’s value chain through accommodating elapsed time between inventory availability and consumption. Inventory plays a very important role in buffering uncertainty in demand and order cycle. It provides protection from uncertainty related to demand fluctuation or unexpected delays in supply. Lastly, inventory compensates for geographical specialisation (Spreading inventory from a central place, and servicing the markets from such central location). It makes it possible for almost every firm or plant to specialize in the product that it manufactures or stores.

2.3.4 The type of inventories

In general there are three main type of inventories (Closs, 2002:13):

- Raw material: used to produce partial products or completed goods. They are made up of good that will be used in the production (e.g sugar, oil, nuts, etc)
- Work in process (WIP): items are considered WIP during the time raw material is being converted into partial product, subassemblies and finished products. WIP occur as result of, for example work delay, long movement time between operations, queuing bottleneck.

- Finished product. This is a product that is ready for current customer sale. It can also be used to buffer manufacturing firms from predictable or unpredictable market demand and seasonal change.

Other categories of inventories should be considered rather from functional standpoint (Shah, 2009:72). These include:

- Stock inventory: the inventory resulting from the production or purchase in batches. The lots are produced and purchased in cyclical lots.
- Buffer/Safety stock: this type of inventory is used as a safeguard against uncertainties of demand and supply.
- Decoupling stock: this type of inventory is held to decouple and separate different parts of a company's operations so that they can function independently from one another.
- Anticipation stock: this is inventory accumulated in advance of expected peak in sale (upcoming season such as Valentine's Day, Easter, Christmas day, etc). Note that failure to sell in anticipation period could be disastrous because the company may be left with considerable amount of stock
- Pipeline /transit inventory: this is inventory on route from one place to another. It may be affected by choosing alternative modes of production or transportation.

Since inventory depends on the specific need of the industry and business, thus inventory found in distribution environment (mainly finished goods for sale) are fundamentally different from those found in manufacturing environment such as raw material and WIP (Senapati, 2012:104). In the world of distribution, retailing, and replacement of parts, organizations deal with finished goods. In the manufacturing world, organizations deal with raw materials and subassemblies. Consideration of what to buy, when to buy, and in what quantities, and so on are totally different in these two worlds. In distribution, one is concerned with having the right item in the right quantity (Senapati, 2012:105). Keeping inventory is not always the good solution because it may leads to wasted money and space. This is due to the fact that the existing formulae used in computing inventory requirement in the distribution system focus on item and quantity rather than place and time.

Because in manufacturing system one is concerned with having the right item in the right quantity, at the right time and in the right place, the target to be reached should be specific. This goal can only be achieved if a strategy that consist of holding inventories in store is effectively implemented.

2.3.5 Reasons for holding inventories

While determining the level of inventory in a supply chain system, it is often assumed that the demand is constant and that the supplier is reliable enough, which means that the quantity ordered is obtained in a given day. Unfortunately, customers do not behave in a predictable way and suppliers also work with production and transportation systems that have some degree of non-reliability. This kind of supply chain environment can be defined as stochastic because the parameters involved vary randomly over time. Therefore, the main purpose of stock is to act as a buffer between supply and demand. This may allow operations to continue smoothly and avoid disruption. Below are some of the reasons why any organization should keep inventory in store (Closs et al, (2002:14) :

- Act as a buffer between different stages of the supply chain
- Allow for demand that are larger than expected, or at unexpected time
- Allow for deliveries that are delayed or too small
- Take advantage of price discount on large order
- Allow the purchase of items when the price is low and expected to rise
- Allow the purchase of items that are going out of production or are difficult to find
- Allow for seasonal operations
- Make full load and reduce transport costs
- Give cover for emergencies
- Can be profitable when inflation is high

The most important reasons for obtaining and holding inventories are further described by Muller, (2003:243):

- Predictability: In order to engage in capacity planning and production scheduling, one has to control how much raw materials, parts, and subassemblies are processed at a given time. Inventory act as a buffer for what different processes need in particular periods.
- Fluctuation in demand: A surplus of inventories on hand is a protection. It is impossible to know how much of a particular good is needed by customers at any given time, but customer demands and production demands still have to be satisfied on time. If it would be possible to see how customers are acting in the supply chain, surprises due to fluctuation in the demand would be held to a minimum.
- Unreliability of supply: Inventory protects from unreliable suppliers or when an item is scarce and it is difficult to ensure a steady supply. Whenever possible, unreliable suppliers should be rehabilitated through discussion or they should be replaced. Rehabilitation can be accomplished through master purchase orders with timed product release, price or term penalties for non-performances, better verbal and electronic communication between the parties, etc. This will result on the lowering of the on-hand inventory needs.
- Price protection: Buying quantity of inventory at appropriate time helps avoid the impact of cost inflation. Note that contracting to assure a price does not require actually taking delivery at the time of purchase. Many suppliers prefer to deliver periodically rather than to ship an entire year’s supply of particular Stock-Keeping-Unit (SKU) at one time.
- Quantity discounts: Often bulk discounts are available if you buy in large rather than small quantities.
- Lower ordering costs: If a larger quantity of item is bought less frequently, the ordering cost is less than buying small quantity repeatedly. However, the cost for holding the item for a longer period will be greater. In order to hold down ordering cost and to adjust to favorable pricing, many organization offer blanket purchase order coupled with periodic release and receiving date of the SKUs called for.

2.3.6 Decisions to make regarding the creation and holding of inventory

There are three basic options to take into consideration regarding the creation and holding of inventory (Hugo, 2003:254):

Cycle inventory - This is the amount of inventory needed to satisfy demand for the product in the period between the purchases of the product. Cycle inventory exists because the production or purchasing in large quantity allows a stage of the supply chain to exploit economies of scale and lower costs. Cycle inventory is the average inventory in the supply chain due to either production or purchases in lot size that are larger than those demanded by the customer are. The primary role of cycle inventory is to allow different stage in the supply chain to produce or purchase product in lot size that minimize the sum of material handling, ordering and holding costs. Note that the quantity of item to order is function of the assumption made regarding demand (e.g it can be evaluated under the assumption of either a deterministic demand or stochastic demand).

Anticipation inventory – This is the inventory that is built up in anticipation of predictable peak in sale that occur in a certain period of time of the year. The alternative to building an anticipation inventory is to set down a flexible inventory management tool that quickly deal with fluctuation in demand and lead time.

Safety stock - Inventory that is held as a buffer against uncertainty. If demand and lead time forecasting could be done with accuracy, then the only inventory that would be needed would be cycle inventory. Since every forecast has some degree of uncertainty, holding additional inventory is therefore necessary in case demand fluctuates over time. Despite the fact that holding some inventories is necessary, one may always think of having enough safety stock so as to satisfy customer demand or finding the probability that a certain level of on-hand-inventory will not lead to stock out. Hence there is the need to set a service factor or level which is the coefficient set to guarantee that the probability of stock out should be small enough.

Managing inventory, especially safety stock, can be considered as one of the most challenging tasks facing supply chain managers (Chopra & mendl, 2007:27). For instance, decisions related to inventory levels throughout the supply chain has a fundamental impact on the service level, response time, delivery lead time and the total cost of the supply chain (Sitompul, 2006:1). Unfortunately, some inventory managers use hunches to set safety stock level, while others base them on a portion of cycle stock level, for example. While easy to execute, such techniques generally result in poor performance. A proposed mathematical approach to safety stock management will not only justify the required inventory levels to business, but also balance the conflicting goal of maximizing customer service and minimizing inventory cost.

From a mathematical point of view, one should see an inventory review system as the relationship that exist between the quantity of stock on order and the quantity available, as pre-set and determined by the individual manager (Ostrow, 2009:214). Although it is not realistic to examine all the possible relationship between the parameters involved in inventory management, it is important to describe those that are most important in selecting an inventory review system. Thus, the relationship that exist between the consumption rate which is dependent on the path of demand and the lead time is useful in that it can help to deeply understand the cost of implementing continuous *(r, Q)* and periodic *(R, S)* inventory review systems.

2.3.7 The relationship between inventory level, consumption rate and lead time

This relationship between inventory level, consumption rate and lead time that is basic to all inventory system is (Whitt, 1988:313).

$$Inventory_{Level} = Consumption_{Rate} * \mathrm{Re}\,sidence_{Time} \tag{2}$$

Where the consumption rate (*flow rate)* is the quantity of stock on order per unit of time and the residence time (lead time) is the total time it takes to deliver the product after an order is given. Equation (2) is called Little’s Law (Whitt, 1988:313).

2.3.8 The inventory cost model

The general cost that result from an inventory can be given as follows (Within & Hadley & Rossetti, 2010:353).

$$Inventory_{Cost} = Cost_{Factor} * Inventory_{Level}(t) \qquad (3)$$

Where the *Inventory Level* can be made up of many components related to the different operations (ordering inventory, holding inventory, back order inventory, etc) involved in the process of managing the inventory system, and the *cost factor* is allocated to each operation. Equation (1and 2), according to the existing theory, illustrate that the cost of implementing an inventory review system implicitly depends on demand and lead time that are known as the main driver of every inventory system (Hadley & whiting, 1976:17).

In general, the main contributing factors to excessive inventories cost, of course, are that lead time and demand, which are not exactly predictable. As a consequence, the primary objective of managing an inventory system should be to find effective ways that can help to model these parameter. Then the next objective, which is the minimization of the inventory cost could be performed accordingly.

The minimization process may compel one to look at the relationship that exist between demand, lead time and the cost of implementing an inventory system and find effective ways to improve it. In fact, there are situations where the randomness of demand and lead time had not been considered under various circumstance, such as poor manufacturing performance, by previous researchers to estimate future demand and lead time (Hsiao, 2008:3). This current study intends to fill the gap.

2.4 Periodic *(R, S)*, continuous *(r, Q)* and a hybrid inventory cost models and issues

Inventory is one of the major sources of cost in the supply chain and it has a huge impact on responsiveness (Muzumdar et al, 2006:4). Implementing inventory review policies on most uncertain manufacturing-company-supply chains tends to be difficult and may lead to considerable cost increase (Merkuryeva & Setamanit, 2011:2). In order to control those costs and act upon them effectively, programs that deal with inventory cost minimization are being used by most inventory managers (Pawlack, 2007:11). The strategy involves defining how often inventory levels must be reviewed to determine when and how much items to order (Rossetti, 2010:36).

The cost of operating an inventory review policy within a supply chain over a period of time is a function of the service level, the demand and the lead time uncertainty (Rossetti & Chopra, 2010:35). Thus, firms that accurately estimate demand and lead time uncertainty, given a specific service level, are able to adapt to changes, satisfy customer demand, minimize their safety stock and inventory cost.

There are two particular angles from which every inventory review policy should be analysed. In reality, the inventory level depends on whether inventories are piled up or shipped out. Most manufacturing companies utilize a piled up (Make-To-Stock) strategy in order to satisfy potential random future demand (Kaminsky, 2009:102). However, in piled up strategy, companies need to be able to estimate demand to determine how much to produce and stock. This system relies on demand which is not most of the time very accurate. Another type of strategy commonly used by distribution firms is the shipped out or Make-To-Order (Kaminsky, 2009:103). In this case, companies produce on the basis of actual customer demand alone. Customers may wait for delivery. This situation can contribute to a loss of competitiveness on the part of the firm. From the above statements, it is clear that the decision to use either a piled up or shipped out strategy for a particular product should be the first activity to consider before choosing the type of inventory review policy as management tool.

Companies often utilize a periodic *(R, S)* and continuous *(r, Q)* inventory review policies in their supply chain systems to determine the appropriate inventory level for ordering purpose. However, both inventory review policies have advantages and disadvantages because they operate differently. Sometimes a way to improve on the two inventory models is to apply the hybrid inventory model. A broadly recognized definition of hybrid systems is still lacking. Many authors consider a hybrid system as a system consisting of at least two interacting subsystems, characterized by continuous and discrete event dynamics (Mosterman, 1999:167). Another definition is provided by (Branicky et al.1998:39) who stated that hybrid systems are systems that combine continuous and discrete inputs, outputs and states or dynamics. Simply put, hybrid systems may involve both continuous-valued and discrete-valued variables and their evolution is described by an equation that generally depends on both (Balbis, 2007:261). A detailed description of hybrid phenomena can be found in other literature (Lunze, 2002:11).

Zhang (2003, 681) proposed a hybrid model that was built from a continuous *(r, Q)* model using both backorders and emergency orders to handle shortages. This model was built assuming stochastic demand and lead time constant. It should be observed that the hybrid models that was built from continuous *(r, Q)* inventory model produced results that can be improved by using a different approach. For instance, very few hybrid inventory models have been derived from both periodic*(R, S)* and continuous*(r, Q)* inventory review policies which is another concerns of this thesis.

The stochastic nature and complexity of Supply chains system are usually considered as an obstacle in the use of mathematical equations (Harkan & Bank, 2007:330). Inventory control problems in the supply chain are solved by using analytical, simulation and hybrid approaches (Pawlack, 2007:12). In order to achieve this goal it is clear that one may be compelled to use appropriate tools.

Many time it is not only desirable but also natural, to use a periodic *(R, S)* or continuous *(r, Q)* inventory review policy alone to deal with the dynamic behavior of a system. However, when both review policies coexist and interact with each other, it is important to develop models that accurately describes the dynamic behavior of such hybrid design (Antsaklis, 2000:879). Hence the need to make an attempt on a hybridization approach that may result from a combination of both inventory review systems.

In this study, one is to design such a hybrid inventory system and test it to see whether it can be used to deal with the disadvantages of implementing either a continuous *(r, Q)* or a periodic *(R, S)* inventory review system. The proposed optimization approach exploit the randomness of demand, lead time under specific circumstance (down time machine) and study their effects on the inventory cost. More attention are then devoted to formulating accurate model for lead time and demand with regard to variation. How to go about assessing the effect of continuous *(r, Q),* periodic *(R, S)* and a hybrid inventory review system on cost while considering appropriate constraint such as customer service and system flexibility is the main focus of this research thesis. The results obtained from this study can help managers to understand that the proposed inventory review models are aiding, and are not meant to replace the one they have been using and that it is for their benefit to incorporate new approaches.

Chapter 3 Research and methodology

3.1 Introduction of methodology

Recall that the main goal of this study is to evaluate the impact of using a flexible periodic *(R, S)*, continuous *(r, Q)* and hybrid inventory review policies on appropriate set of supply chain performance measures, with the aim of optimizing each of the models.

The following sub-headings explain how the research main objective and specific objectives were achieved: research locale; research design; data-collection; inventory model formulation and development, a case study simulation-optimization for the three inventory review models.

3.2 Research Locale

A company in the Vaal region is among those enterprises that were facing the problem of meeting the required inventory level for some of the products in its supply chains. Past data coming from that manufacturing company around the Vaal region were used.

3.3 Research design

According to Wilson (2010:102), a case study strategy is a detailed framework or plan that helps to guide one through the research process, allowing a greater likelihood of achieving research objectives. It is also defined as an in depth description of an individual, group or organization, either for the purpose of testing whether the case fits to a specific theory, or fits one theory better than another, or to simply determines what makes the particular case, inferior, superior, or different to other similar cases (Yin, 1994:12). For this kind of research study, adopting a comparison perspective, a case study strategy was considered as an appropriate choice of methodology.

Although other approaches can be used, one should note that they are not closely aligned with the aim of this study. For instance, this study did not aim to change practice as would have been the case with others strategies. Instead, the aim was to design study that would help address the research question in a different and effective approach. Adopting a case study coupled with an optimization simulation approach as method had allowed one to set down a strategy that could control and manipulate variables, and then provide an answer to the research question.

Since the research question in this study relates to the understanding of the relationship between the inventory model's input parameters and the inventory cost, the unit of analysis adopted for this study is:

- The relationship between the order quantity, the re-order point (or re-order period) in achieving the optimal inventory cost for a continuous *(r, Q)*, periodic *(R, S)* and hybrid inventory review system in the company's supply chain.

3.4 Case study company background

A case study company is a manufacturing firm around the Vaal region which produces and sells more than 2 chemical products all over the country. There, products are distributed via company owned distribution. More frequently, the distribution center orders products from the factory, stores them at its facility, and waits for customers (agents or retailer), who then sell the products to their own customers. One of the products that are distributed and that represent the majority of annual sale is analysed. The current inventory policy being used is periodic review *(R, S)*. At the beginning of each week, the inventory manager of the distribution center reviews weekly sale forecast together with the space available in the distribution center and decide on the quantity to order by using his own experience. Further, the inventory managers order a certain amount of product such that the high level of inventory to prevent stock-out problem be maintained.

For this study, the company would like to further examine other possibilities such as a flexible periodic *(R, S)*, continuous *(r, Q)* and a hybrid inventory review systems in order to identify the best one. In addition, the company also would like to better understand the impact of different factors (random demand and lead time) on cost while not violating a specified service and system's flexibility level. Simulation optimization method was used to identify the optimal values for re-order point (s), review period and order quantity. In order to achieve these goals, data required for analysing the system and making better decision on its inventory are found in section appendix.

3.5 Inventory Models conceptualization

Please note that the logic around the derivation of most of the parameters meant for continuous review policy are similar to those for periodic review policy *(R, S)*. As such, all the re-derivations were not repeated under the periodic and hybrid review subheadings. Only the differences between the three review policies were presented under the periodic and hybrid review subheading.

3.5.1 Continuous (r, Q) inventory review problem

In a continuous *(r, Q)* inventory review system, the stock level of a single item is observed continuously. An order of size *Q* is placed whenever the inventory level drop below the re-order point *r*. The ordering quantity *Q* is received after a given Lead Time *L*. The demand that cannot be satisfied directly from the stock is backordered. Such inventory review system is, thus made up of the activities such as ordering, holding and backordering. It follows that the general expression of the annual cost of ordering, holding inventory and incurring backorder that can be used as the basis of the analysis may be described as follows (Rossetti, 2010:354)

$$TC(r,Q) = k * N(t) + h * \frac{1}{T}\int_0^T I(t)dt + j * \frac{1}{T}\int_0^T B(t)dt \tag{4}$$

Where *TC* is the total cost per unit time of implementing a continuous *(r, Q)* inventory review policy, *k* is the order preparation cost per order, *N(t)* is the number of replenishment orders made per unit time, *h* is the holding cost for the item in units per time, *T* is the period or cycle time, *I(t)* is the instantaneous inventory on hand, *j* is the backordering cost for the item in unit per time and *B(t)* is the instantaneous inventory backordered. It should be noted that the number of replenishment per time *N(t)* is dependent on the demand rate and the instantaneous inventory on hand. *I(t)* is dependent on depletion rate. Both *I(t)* and *B(t)* are function of demand and lead time. In order to relate expression (4) with the empirical or practical observations, the cyclical cost of ordering inventory, holding inventory and incurring backorder as described in equation (4), have to be realistic, which are achieved by properly considering Demand and Lead Time which are two important parameters of this continuous *(r, Q)* inventory review system. The next paragraphs deal with this issue.

The model for stochastic Demand that should allow for the variability can be given as follows (Makridakis & Gardinier, 1989:602).

$$\partial D = D_{rate}dt + f_C dW(t) + f_P dV(t) \tag{5}$$

Where D_{rate} is the incremental change in demand within infinitesimal time interval $[t, t+dt]$, more commonly called drift term in stochastic language such that $dt \ll 1$, f_c is a continuous fluctuation term, f_p is the jump term that reflects the magnitude of instantaneous jump in demand, $dW(t)$ is increment of Weiner Process and $dV(t)$ is the number of stochastic counting process or number of jump changes in demand with the infinitesimal time interval. Note that the parameters f_c and f_p should be chosen so that ∂D (i.e expression (5)) correlates with (or reflects) real life situation.

Another interesting issue to be dealt with involves identifying the stochastic nature of the lead time. This is achieved by proposing a stochastic counterpart of the relationship between Lead Time, lot-size, cycle time, set-up time and non-operation time. This deterministic relationship can be written in general as follow (Groover, 2001:71).

$$MLT(t) = n_0\left(T_{SU} + Q * T_C + T_{no}\right) \qquad (6)$$

Where MLT = Lead Time for an order, n_0 is the number of "machines" or departments along the inventory-replenishment-line, T_{SU} is the up time or waiting time to start preparing the lot-size, Q is the order Quantity which may vary randomly as demand fluctuates, T_C is the cycle time or operation time, T_{no} is the non-operation time (i.e. source of before-production-delay) mainly due to unnecessary material handling or transportation, waiting, queuing, etc. From the general model described in equation (6), the MLT expression that depends on the three types of maintenance program (i.e stochastic disruptions) are proposed as follows.

Reactive maintenance is the type of maintenance in which equipment are maintained when they break down or which occurs within production cycle. This mode of maintenance is still being preferred and employed by some African manufacturing companies since it requires less staff and does not incur capital cost until something breaks. In this case, the Lead Time may be represented by.

$$MLT(t) = n_0 * \left(T_{SU} + Q * \left(T_{Operation} + F_{Corrective} * T_{Corrective}\right) + T_{no}\right) \tag{7}$$

Preventive maintenance program is the type of operation in which maintenance actions are performed according to a specific schedule so as to detect, prelude or minimize the degradation of equipment, or extend their life through controlling their degradation to an acceptable level. Since preventive operation is performed while equipment is at rest, the Lead Time is given as:

$$MLT(t) = n_0 * \left(T_{SU} + Q * \left(T_{Operation}\right) + F_{\mathrm{Pr}eventive} * T_{\mathrm{Pr}eventive} + T_{no}\right) \tag{8}$$

Predictive maintenance is the type of operation in which measurements that detect the beginning of equipment degradation are taken so that they can be eliminated or controlled in either reactive or preventive maintenance. In predictive maintenance, the Lead Time is, thus, given as follow:

$$MLT(t) = n_0 * \left(T_{SU} + Q * \left(T_{Operation} + F_{Corrective} * T_{Corrective}\right) + F_{\mathrm{Pr}eventive} * T_{\mathrm{Pr}eventive} + T_{no}\right) \tag{9}$$

For all the three types of maintenance program mentioned above, the probability of breakdown or downtime is represented by F_i and the downtime is represented by T_i where i stands for the type of maintenance operation (i.e. preventive, corrective and predictive).

The time increment of *MLT* may then be obtained and results made stochastic by addition of periodic and continuous fluctuation terms given as follows (Iwankiewicz, 1999:34)

$$d(MLT(t)) = T_C * \frac{\partial Q}{\partial t} dt + Fluctuation \tag{10}$$

After having proposed stochastic models for Lead Time and Demand, the next step is to propose the general cyclical cost of ordering inventory, holding inventory and incurring backorder as described in equation (4) for a continuous *(r, Q)* inventory review models.

3.5.1.1 Number of replenishment $N(t)_{con}$

Let *D(t)* be the random Demand that occur within an interval *(0,T]*, and *Q(t)* is the order quantity each time an order takes place, then order frequency of replenishment over this period of time *T* is referred to the number of replenishment orders made per time (*N(t)*, which can be given by

$$N(t)_{cont} = \frac{D(t)}{Q(t)_{con} * T} \tag{11}$$

3.5.1.2 Inventory held $I(t)_{con}$

In a continuous *(r, Q)* inventory review system, the value of inventory *I(t)* per cycle consists of two parts: safety stock inventory denoted I_B and the consumption inventory, denoted I_A. With reference to what has just been said, a possible instantaneous consumption inventory model can be described as follow

$$I_A = (Cycle_{Stock}) = \frac{1}{T}\int_0^T \left(I_0 - \left(\frac{\partial D}{\partial t} \right) * t \right) dt \tag{12}$$

Where I_0 is the initial inventory, demand rate or depletion rate is $\partial D/\partial t$ and *t* is the consumption time and *T* is the inventory cycle stock period. Note that each time a particular demand occurs, the inventory level is reviewed to check whether the re-order point has been reached or not. As a result of the consumption inventory, the safety stock inventory is presented as follow

$$I_B = \left(r - \left(\frac{\partial D}{\partial t} \right) * \left(MLT(t) \right) \right) \tag{13}$$

Where the re-order point is *r*. Putting together, it is easy to see that maximum instantaneous inventory held per cycle equates to

$$I(t)_{cont} = I_A + I_B \tag{14}$$

3.5.1.3 Inventory backordered $B(t)_{con}$

The general expression of the instantaneous backorder per time can be given by.

$$B(t)_{cont} = \int_0^T \left(B_{bo.cont} * T_{b.cont} \right) dt \tag{15}$$

Where the

$$B_{bo.cont} = \frac{1}{period} \int_0^T \left[\frac{N_{bo}}{N(t)_{cont}} \right] dt,$$

Where $B_{bo.cont}$ is the backorder rate, $T_{b.cont}$ is the time length of the backorder, N_{bo} is the number of back order at time t and $N(t)_{cont}$ is the number of order per period under a continuous review policy (r, Q) and $N_{bo} / N(t)_{con}$ is the stock-out probability.

Recall that in continuous review policy *(r, Q)*, it is inevitable that shortage takes place with the assumption of stochastic lead time and demand. In such practical situation, the shortage cost can be difficult to estimate. For this reason, managers often decide to control shortage on the basis of the service level (Rossetti, 2010: 234). The service level is given by

$$Service_{Level} = 1 - \frac{N_{bo}}{N(t)_{con}} \tag{16}$$

Let $T_{b.cont}$ (t) be the time length for the backorder quantity during a period. Then, $T_{b.con}$ (t) can also be given by

$$T_{b.cont}(t) = MLT(t)_{BO} + PROD(t)_{BO} \tag{17}$$

Where $MLT(t)_{BO}$ is lead time for backorder delivery, $PROD(t)_{BO}$ is production time for backorder.

Recall that the goal is to minimize equation (4) while considering appropriate constraint such as customer service and system flexibility. A flexible inventory system is defined as a system that demonstrate its ability to meet random fluctuation in demand and delivery time. To find the value of the decision variables r and Q that minimize the cost of implementing such a continuous inventory review system $TC(r, Q)$, it is derived from the first order condition (F.O.C) for each variable by taking partial derivatives and solve them simultaneously (see equation 18 below and remember that ∂ denotes partial derivatives):

$$\begin{cases} \frac{\partial_{TC(r,Q)}}{\partial_{Q}} = 0 \\ \frac{\partial_{TC(r,Q)}}{\partial_{r}} = 0 \\ d_{TC(r,Q)} = \frac{\partial_{TC(r,Q)}}{\partial r} dr + \frac{\partial_{TC(r,Q)}}{\partial_{Q}} d_{Q} = 0 \end{cases} \tag{18}$$

3.5.2 Periodic inventory review *(R, S)* problem

Under a periodic inventory review system (R, S), the level of inventory is checked after some given and known time interval R called the replenishment period. Then, an order is placed to bring the inventory level up to S. The quantity ordered is equal to the difference between S (order-up-to level) and instantaneous inventory level $I_A(t)$. It should be noticed that if the inventory is at level $I_A(t)$, the order quantity $Q_p = S - I_A(t)$ is ordered to bring the inventory position to S.

For the sake of the study, the patterns and models of Demand and Lead Time are the same as described in equation (5), (6), (7), (8) and (9). The same inventory cost model development as the one used in a continuous (r, Q) inventory review system is applied in this section. However, one has to specify that from the operational point of view, in the periodic inventory review system (R, S), both the review period and the order quantity can vary according to need. Thus, the decision variables in periodic review system are review period R and order quantity Q_p, while those in the continuous review period are re-order r point and re-order quantity Q_c.

3.5.2.1 Number of replenishment $N(t)_{per}$

Let *D(t)* be the random Demand that occur within an interval *(0,T]* and *N(t)* the number of replenishment or number of order placed within the mentioned interval, Q_P is the order quantity, $T_L(t)$ the consumption time protection interval and finally *R* the review interval.

The number of order placed for the periodic inventory review system *(R, S)*, is computed the same way as with a continuous *(r, Q)* inventory review system, provided that the computation of the order frequency should be expressed as function of demand and the review period.

$$N(t)_{per} = \frac{D(t)}{Q_P(t)_{per} * T} \tag{19}$$

3.5.2.2 Inventory held $I(t)_{per}$

In order to determine the inventory held under a periodic inventory review system *(R, S)*, the same principles as the one used with a continuous *(r, Q)* inventory review system are applied in this section. However, the value of inventory before the replenishment time (safety stock), and after the replenishment time (cycle stock or inventory on hand) are computed with respect to the stochastic protection demand interval. From an operational point of view, the protection demand interval or total lead time under a periodic (R, S) system is given as follow (Drake, 2008:123).

$$T_L(t) = R + MLT(t) \tag{20}$$

Where T_L(t) is the protection Demand interval or total consumption time. The consumption inventory at review point is given as function of time since replenishment *R* may vary with demand.

$$I_{A2} = \left(\frac{1}{T} \int_0^T \left(I_0 - \left(\frac{\partial D}{\partial t} \right) * R \right) dt \right) \tag{21}$$

While the safety stock inventory can be described by the following expression.

$$I_{B2} = \left(S - \left(\frac{\partial D}{\partial t} \right) * T_L(t) \right) \quad (22)$$

Hence the total instantaneous inventory held

$$I(t)_2 = I_{A2} + I_{B2} \quad (23)$$

3.5.2.3 Inventory backordered $B(t)_{per}$

Note that the general expression of the instantaneous backorder inventory per time was computed as equation (14, 15 and 16). Therefore, the general expression of the instantaneous backorder inventory per time can be given by.

$$B(t)_{Periodic} = \int_0^T \left(B_{bo.periodic} * T_{B.per} \right) dt \quad (24)$$

Where

$$B_{bo.per} = \frac{1}{period} \int_0^T \left[\frac{N_{bo}}{N(t)_{per}} \right] dt, \quad (25)$$

Where $B_{bo.per}$ is the periodic backorder rate, $T_{b.per}$ is the time length of the periodic backorder, N_{bo} is the number of back order per time and $N(t)_{periodic}$ is the number of order per period under a periodic review policy, $N_{bo} /N(t)_{per}$ is the stock-out probability or service level.

Let $T_{b.per}$ (t) be the time length for the backorder quantity during a period. Then, $T_{b.per}$ (t) can also be given by (see equation 17).

$$T_{b.per}(t) = T_{b.cont}(t) \quad (26)$$

$$T_{b.per}(t) = MLT(t)_{BO} + PROD(t)_{BO}$$

To find the value of the decision variables *R*, *S*, and Q_P that minimize the cost of implementing a periodic inventory review system, the following expression are used.

$$\begin{cases} \dfrac{\partial_{TC(R,Q_P)}}{\partial_R} = 0 \\ \dfrac{\partial_{TC(R,Q_P)}}{\partial_{Q_P}} = 0 \\ d_{TC(R,Q_p)} = \dfrac{\partial_{TC}(R,S)}{\partial R} dR + \dfrac{\partial_{TC}(R,C)}{\partial Q} d_{Q_p} = 0 \end{cases} \qquad (27)$$

3.5.3 Hybrid inventory review problem

Here, an order to replenish the system should then be placed if the inventory level drop below a specified level before a particular review date, if not, the order quantity is determined at the next review date. One may easily see the need of combining feature of a Periodic *(R, S)* and Continuous *(r, Q)* inventory review system.

Such hybrid inventory system can further be described. Note that as demand occur in time, the consumption or utilization inventory may either be continuously depleted from the initial inventory or change discontinuously in response to control command or event. Such command may be triggered by marketing strategy such as promotion and equipment downtime which in turn may cause the inventory system to be out of stock before the review date *R*. In order to reduce these number of stock-out (backorder) that may occur, in order to replenish the system immediately (jump inventory) can be allowed. A mathematical description of this hybrid inventory system is found in the next sections.

3.5.3.1 Number of replenishment $N(t)_{hybrid}$

The features needed to describe Periodic*(R, S)* and Continuous *(r, Q)* review combination system are to be used to determine the number of replenishment over the period. This hybrid system may depends on the relative numbers of replenishment performed under both review systems. Let $N(t)_{con.h}$ be the number of replenishment performed per period under a Continuous *(r, Q)* review system and $N(t)_{per.h}$ the number of replenishment performed per period under a Periodic *(R, S)* review system needed for hybrid model at a specified capacity utilization level *U.* Represented as a percentage, the capacity utilization is the extent to which the productive capacity of a firm is being used in generation of goods and services. It is then possible to formulate the equation for the total number of replenishment in performing both Periodic *(R, S)* and Continuous *(r, Q)* review system as follows (Groover,2008:348).

$$N(t)_{Con_h} * T_{Cont_h} + N(t)_{Per_h} * T_{Per_h} = T * U \qquad (28.1)$$

$$N(t)_{Con_h} * Q_{Cont_h} + N(t)_{Per_h} * Q_{Per_h} = D_{Total} \qquad (28.2)$$

$$N(t)_{Cont_h} + N(t)_{Per_h} = N(t)_{Hybrid} \qquad (28.3)$$

Where *U* is the capacity utilisation during the period in percentage, *T*U* is the total time (number of weeks) of operation per period, $T_{cont.h}$ is the replenishment cycle time under a continuous *(r, Q)* review system in weeks/year, $T_{per.h}$ is the replenishment cycle time under a periodic *(R, S)* review system in week/year, $Q_{con.h}$ is order quantity under a continuous *(r,Q)* review system needed for a hybrid model, $Q_{per.h}$ is the order quantity under a periodic *(R,S)* review system needed for a hybrid model, $N(t)_{hybrid}$ is the total hybrid inventory replenishment.

In this case, the cycle time (time/cycle) under a periodic *(R, S)* inventory system can therefore be expressed by

$$T_{\Pr ev} = \frac{(Q_p)}{\left(\frac{\partial D}{\partial t}\right)} \tag{29}$$

Where Q_p or (Q_{per}) is the order quantity following periodic review system. Further, the cycle time under a continuous (r, Q) review system is.

$$T_{con} = \frac{(Q_c)}{\left(\frac{\partial D}{\partial t}\right)} \tag{30}$$

Where Q_c or (Q_{cont}) is the order quantity following continuous review system.

3.5.3.2 Inventory held $\mathbf{I(t)_{hybrid}}$

From a mathematical perspective, the total hybrid inventory level may be described by three components: the normal consumption inventory, drop inventory and lead time safety stock inventory. The first one, called "the normal consumption inventory may be described as the inventory that is depleted from the initial stock. A possible model is then

$$\begin{aligned} I(t)_{A3} &= \int_0^T (I_0 - \mu(t))dt \\ I(t)_{A3} &= \int_0^T \left(I_0 - \left(\frac{\partial D}{\partial t} * t \right) \right) dt \end{aligned} \tag{31}$$

Where, $I(t)_{A3}$ represents the instantaneous inventory before the re-order (jump process) takes place with, t ($t > 0$) is the time that elapse between the initial time (beginning of operations) to the re-order time, I_0 is the initial inventory, $\mu(t)$ is the degradation/utilization.

The second type of inventory, called “drop inventory” is the one needed to deal with stock-out events when the on-hand-inventory drops below a specified point before the review date *R* as a result of external factors such marketing and promotion. Note that the quantity needed to cover the expected need (demand) at this particular time can automatically be placed. It implies that there should be restocking times t_r and associated order amounts Q_r. This drop inventory performed through the spontaneous replenishment process may be described by

$$I(t)_{B3} = \frac{1}{T}\int_0^T \left(\delta(t - t_r) * Q(t_r)\right)dt \tag{32}$$

Where δ is the Dirac delta function or step function, t_r is the time at which the jump process is observed and Q_r is the order quantity at time t_r.

The third inventory known as safety stock inventory is the one required for dealing with uncertainty during restocking or replenishment. This safety stock inventory is given by

$$I(t)_{C3} = \left(S_{hybrid} - \left(\frac{\partial D}{\partial t}\right) * T_L(t)_{hybrid}\right) \tag{33}$$

Where S_{hybrid} is the target hybrid inventory level, $T_L(t)_{hybrid}$ is the consumption time or protection demand interval. Hence the instantaneous hybrid inventory held

$$I(t)_3 = I(t)_{A3} + I(t)_{B3} + I(t)_{c3} \tag{34}$$

3.5.3.3 Inventory backordered $B(t)_{hybrid}$

It should be noticed that the hybrid backorder inventory can be computed by applying the same principle as for the Periodic *(R, S)* and Continuous *(r, Q)* inventory review systems. Therefore, the general expression of the instantaneous backorder inventory per time can be given by.

$$B(t)_{Hybrid} = \int_0^T \left(B_{bo.hybrid} * T_{B.hybrid}\right) dt \tag{35}$$

Where

$$B_{bo.Hybrid} = \frac{1}{period} \int_0^T \left[\frac{N_{bo.hybrid}}{N(t)_{Hybrid}}\right] dt\,, \tag{36}$$

Where $B_{bo.hybrid}$ is the hybrid backorder rate, $T_{b.hybridt}$ is the hybrid time length of the backorder, $N_{bo.hybrid}$ is the practical number of back order per time and $N(t)_{hybrid}$ is the ideal number of order per period under a hybrid review policy, $T_{bo.hybrid}$ (t) is the time length for the hybrid backorder during a period. Then, $T_{bo.hybrid}$ (t) can also be given by

$$T_{bo.hybrid.}(t) = MLT(t)_{BO\ hybrid} + PROD(t)_{BO_{hybrid}} \tag{37}$$

Where $MLT(t)_{BO.hybrid}$ is lead time for backorder, $PROD(t)_{BO.hybrid}$ is production time for backorder. The value of the decision variables *r, R, S* and Q_P were then obtained by solving appropriate differential equations.

To find the value of the decision variables *r, R* and Q_p that minimize the cost of implementing a hybrid inventory review system, the following expression are used.

$$\begin{cases} \dfrac{\partial_{TC_{Hybrid}}}{\partial_{Q_P}} = 0 \\ \dfrac{\partial_{TC_{hybrid}}}{\partial_R} = 0 \\ \dfrac{\partial_{TC_{Hybrid}}}{\partial_r} = 0 \\ d_{TC} = \dfrac{\partial_{TC\,hybrid}}{\partial r} + \dfrac{\partial_{TC\,hybrid}}{\partial R} + \dfrac{\partial_{TC\,hybrid}}{\partial Q} = 0 \end{cases} \qquad (38)$$

Next step is to analyse the simulation output. The output or performance variables of simulated systems are estimated by using numerical measures of the descriptive statistic such as mean, standard deviation and median (Setamanit, 2011:4). To obtain these estimates with a specified precision, a number of simulation runs, replications and/or the length of the run need to be defined.

In this thesis, the simulation output results are consolidated in data reports and presented by graphs. A graphical comparison is performed because it can allow for making a subjective decision about the model behaviour (Sargent, 2010:130). Further, the use of a graphical analysis do not require assumptions necessary to introduce interval estimation. This approach is preferred because the usage of graphs can appear more transparent and allow for easy interpretation when comparing with data reports (statistical analysis).

Once one has a working optimization simulation approach, the next step is to check that the simulation is actually doing what one expects. The process of checking that the models proposed do what they were planned to do is known as verification. One of the verification techniques suggested by Law (2007:67), such as running the models under simplify assumption and under a variety of the input parameters was used to ensure that the models perform correctly.

3.6 Models verification and validation

While verification concerns whether the models are working as expected, validation deals with whether the simulation is a good model of the target. In order to achieve those goals, results of the models proposed were compared against real company performance.

The process of verifying the proposed models included two steps. Firstly, it was essential totest different scenarios of extreme situations where the outcomes are easily predictable. In other words, random input variables were introduced in the models and the outputs checked later. Constant values of lead time and demand were used in order to evaluate the effect on re-order point r, order quantity Q_c and the inventory cost under a continuous *(r,Q)* review system. The same principle was applied under a periodic review *(R,S)* system in order to evaluate the effect on the review period R, order quantity Q_p and the inventory cost. To make this easier, it was, secondly, desirable to have a system that will automatically run these tests, record the outputs, and highlighting differences between the outputs of the models.

It was found that the model results correlate with the real company data (the results showed that proposed models did what they were expected to do). In fact, the difference observed in the costs of operating both review policies in uncertain supply chains are associated to their parameters (e.g protection demand intervals and order quantity), which of course were predicted in the background section. Some of the results related to the verification process are found in appendix (6). It should be noticed that all the proposed models are able to give insights although the difference between simulation output and data from the target (real company performance) are large. Doubts should not be thrown on the models because they may be correct, but rather to the operational difference that exist between Periodic *(R, S)* and Continuous *(r, Q)* inventory review systems. It should be further noticed that some results that confirm the validity of the proposed models are displayed in chapter 4.

3.7 Data Collection procedure

Page et al, (2003:12) declare that, in most cases, the final theoretical framework for research project rests on two types of data, namely primary and secondary data. The difference between primary and secondary research data collection is that primary research data collection involves conducting research oneself, or using data for the purpose it was intended for. Secondary research data, on the other hand, is collected by a third party or for some purpose. One of the main drawback of secondary data is that the data have not been generated specifically to address the research problem and may therefore contain potential biases or may be unrepresentative of the population in which the researcher is interested in. In this thesis, the question of either using primary data or secondary data was dealt with. Because of the nature of inventory problem addressed, it was found necessary to collect and use primary data as part of the case study.

3.7.1 Case study background

It is important to describe the condition under which the data were collected. A case study company is a manufacturing firm around the Vaal region which produces and sells more than two new chemical products all over the country. There, products are distributed via company owned distribution. More frequently, the distribution center orders products from the factory, stores them at its facility, and waits for customers (agents or retailer), and then sell the products to customers. One of the products that is distributed and represents the majority of annual sale is analysed. The current inventory policy used is periodic review *(R, S)*. At the beginning of each week, the inventory manager of the distribution center reviews weekly sale forecast together with the space available in the distribution center and decides on the quantity to order by using his own experience. Further, the inventory manager orders a certain amount of product so that the high level of inventory to prevent stock-out problem be maintained.

For this study, the company would like to further examine other possibilities such as a flexible periodic *(R, S)*, continuous *(r, Q)* and a hybrid inventory review systems in order to consider using the best one. In addition, the company also would like to better understand the impact of different factors (random demand and lead time) on cost while not violating a specified service and system’s flexibility level. Simulation optimization method was used to identify the optimal values for re-order point (s), review period and order quantity. In order to achieve these goals, data required for analysing the systems and making better decision on its inventory were needed.

In this case study, data, particularly for 2012 and 2013 from a business within the Vaal region was collected. It should be noticed that the study deals with information related to new chemical product that were at the growth phase. Therefore, it was found useful to rely on data collected over a very short period of time. That is why a period of two years of data was chosen as the sample. It was easy to see that weekly past data on demand, lead time, maintenance and inventory were poorly recorded. These data collected were often subject to omission and misinterpretation. Therefore, they could not be used immediately to address the research problem. It turned out that the available data needed a special treatment. Otherwise, they could not really be needed because of their inability to fit into real model's logic. For these reasons, action (regression analysis on the data) was taken to correct the error or variation. What has been done to limit the error that might have occurred as result of data gathering procedure is dealt with in the next section. The data required and collected for this research project are presented as follow.

3.7.2 Demand data

Demand data were collected for two years. Although fully represented in appendix 2 and 3, it was found useful to present these data in figure 2.

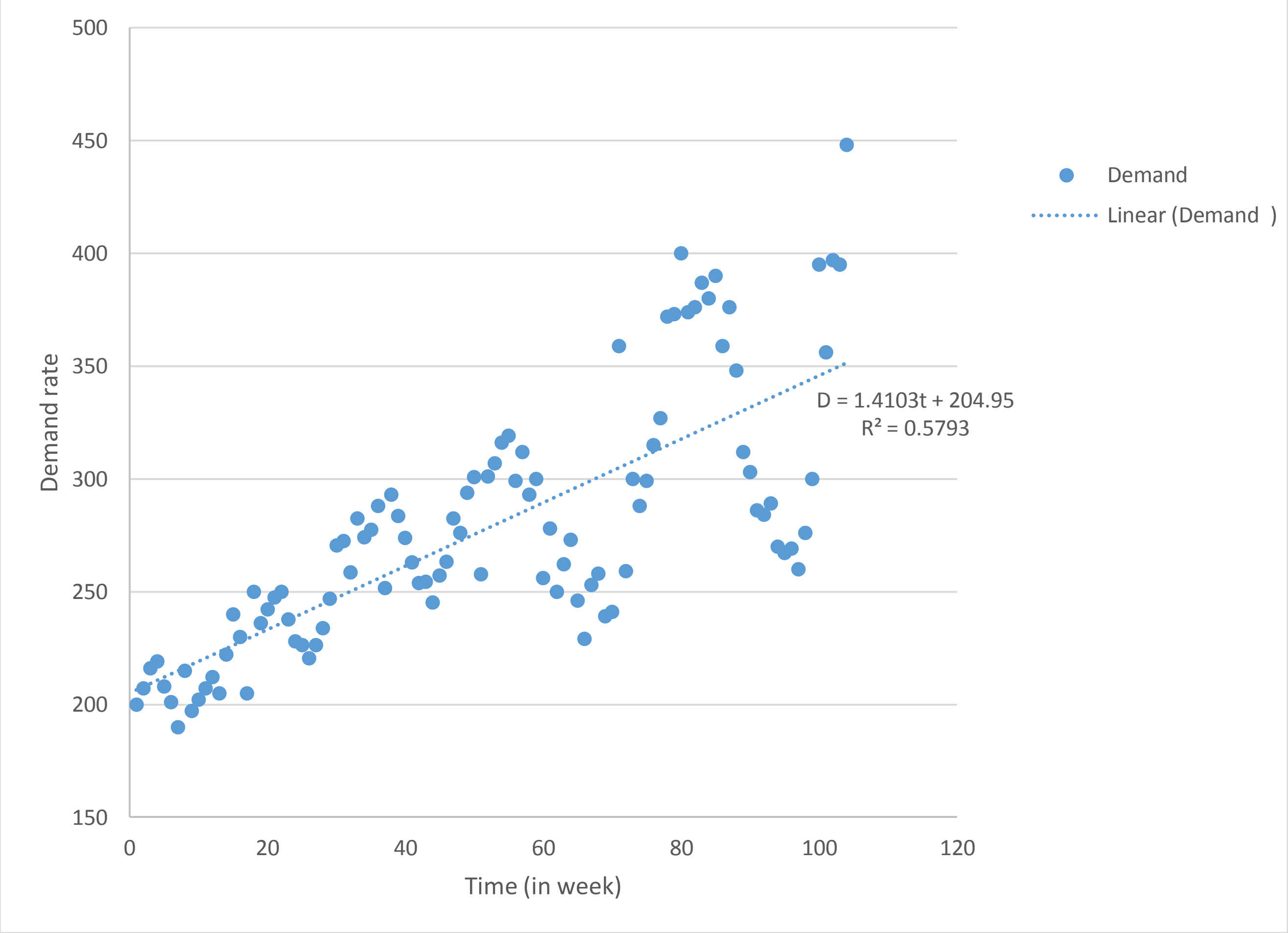


Figure 2 Scatter diagram of demand over 2 years

The scatter diagram shows an increasing trend of demand with fluctuations. It was clear to understand that the component of the demand function that resulted from the collected data could then be generalised as in equation (5):

$$\partial D = D_{rate} dt + f_C dW(t) + f_P dV(t)$$

Where

- Demand rate is $D_{rate} = \frac{\partial_D}{\partial t}$
- Continuous fluctuation term is f_C
- dW(t) is increment of Viener process
- Periodic fluctuation term is $f_p = d * (1 - Sin(2 * t))$
- dV(t) is number of stochastic counting process
- Maximum change in demand due to periodic fluctuation *d*=37

Regression analysis was performed to get some of the parameters of the demand fluctuation model. The statistical value was $R^2=0.6$ that showed good fit of the experimental data by the regression method.

3.7.3 Lead time data

Weekly components (parameters) of the lead time were collected for two years. The lead time data were treated the same way as with demand data. Figure 3 shows the lead time pattern over two years. It should be noticed that a regression line on lead time data was performed.

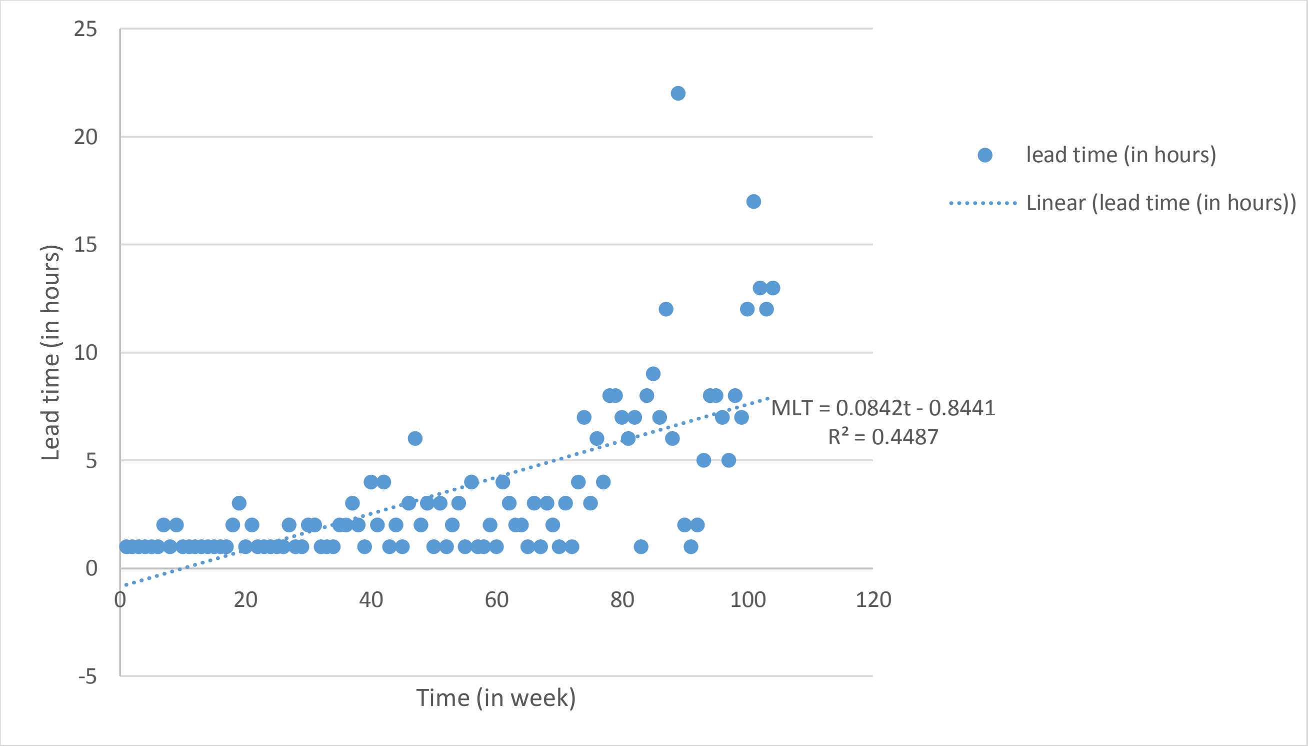


Figure 3 Scatter diagram of the lead time over 2 years (in week)

The lead time function was obtained with statistical measure $R^2 = 0.45$ that capture the percentage of error as explained by the regression model. This result implied that the relationship between total lead time and time was not that much strongly linear. Which was a good reason to think of modelling the lead time in a different way so that it incorporate important parameters that may capture the uncertainty caused by machine downtime.

3.7.4 Inventory management data

Others important parameters of the mentioned inventory review system were obtained from document and archival record (see appendix 3). They are represented as follow:

- The ordering cost *k=R125 per order*,

- The holding cost *h=R0.511 per unit per time*,
- The back order cost *j=R201per unit per time*,
- The service level was set to 90%
- Working hour factor FW = 1/(5*8*104)

Chapter 4 Finding and discussion

In this chapter, the simulation output of the proposed models over two years are presented and discussed. The output results obtained from simulation optimization for each of the three inventory review system are presented and discussed in the next sections.

4.1 Simulation outputs for Continuous *(r, Q)* inventory review system

Recall that one of the specific objective consists of assessing the company annual total cost and stock out probability when implementing the continuous *(r, Q)* inventory model in an uncertain supply chain. Figure (2), reveals the path of demand rate, continuous inventory cost and service level.

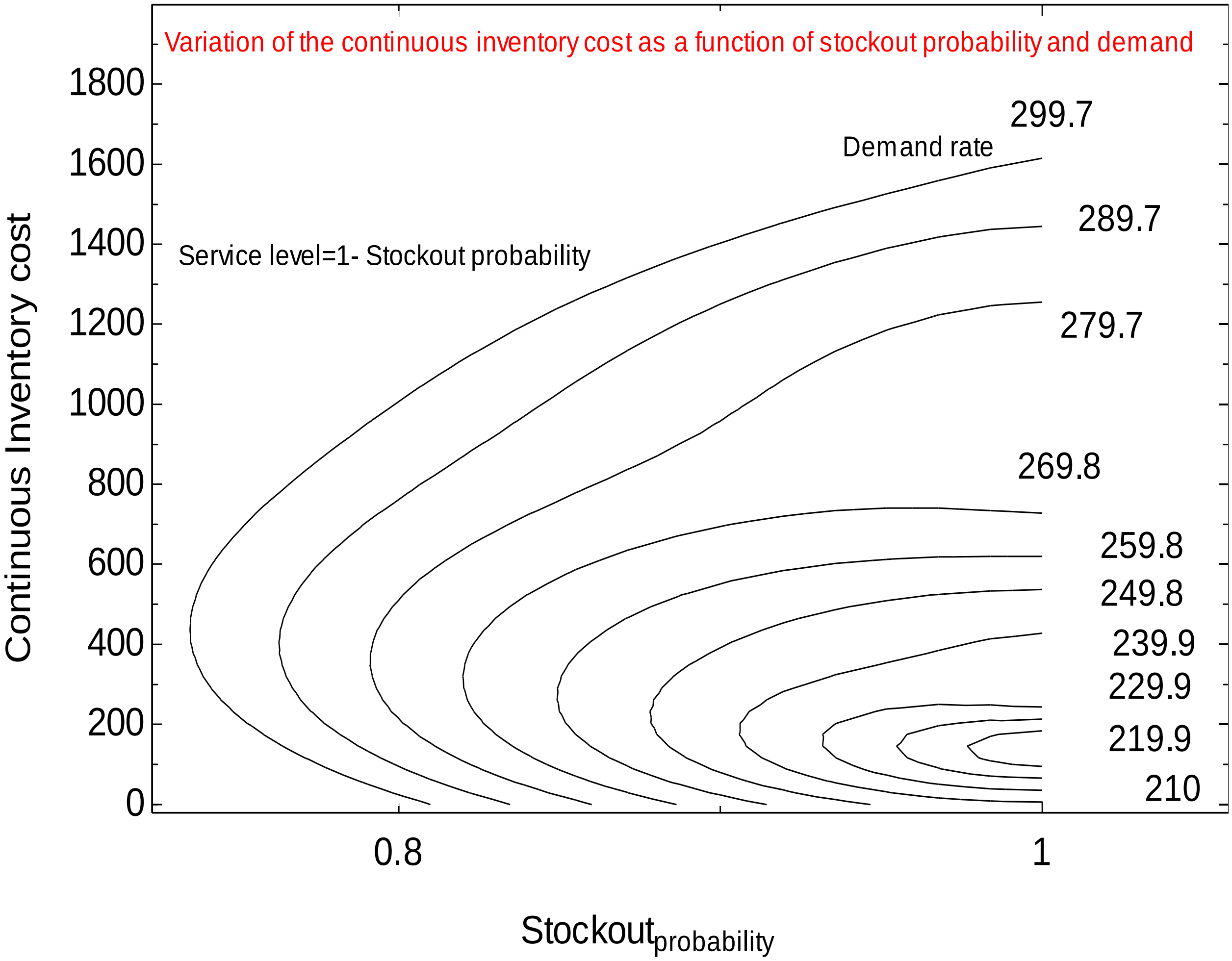


Figure 4: Trend of the demand rate, stock out probability and continuous inventory cost in a stochastic supply chain

As it can be seen from figure 4, that stock out events are allowed to a certain extent. When demand rate is low, stock-out probability is high with low cost. This implies low service level at low demand rate when the amount of money invested on inventory management is low. At the case study company, this fact was observed and the reason given by the inventory personnel was that it was due the fact that there was improper housing keeping and less sorting of product into category by volume, leading to poor inventory management. It was also observed that at constant demand, the cost increased with increase in service level reaching a maximum, where the cost continues to increase with a decrease in service level. This implies that there is a threshold service level that could be obtained with increase cost. This is typical practical observation as it was explained by the inventory manager that increasing cost through ordering and holding inventory reduces backorders, and there reach a point where continuous ordering and holding of inventory lead to a situation where there is extension of review interval and reduction of re-order point, thus leading to increasing stock out. Of course, increase in stock-out implies decrease in service level

Also, as demand rate gets high, service level decreases with increased cost. It should be noticed that higher service level in response to an increasing demand leads to very high inventory cost. The fact that the stock-out probability increases due to demand is probably a reflection of the inventory cost under a backorder case. It should be noticed that the system is likely to be out of stock as demand increase rapidly. This result would suggest that an appropriate amount of inventory must be kept in store if one is to avoid customer dissatisfaction and loss of income. This can also be linked to the work of (Wei-Min, 2012:2) who stated that in a stochastic supply chain system, the effect of demand on the cost and service level (stock out probability) are stronger.

What this finding also portrays about the trend of the inventory cost, stock out probability in an uncertain supply chain is that implementing a continuous *(r, Q)* inventory review system would incur higher cost over time. This finding can be used as performances indicator because it may then be a proof that one should start investigating on the relationship that exist between the inventory cost and other appropriate parameters in order to make decision. The plots displayed in the next sections address this issue as per the following specific objectives:

- Study the evolution of optimum re-order point and order quantity (lot-size)
- Measure the effect of changes in re-order point and order quantity on the inventory cost of implementing a flexible continuous *(r, Q)* inventory review system

Before addressing the above specific objectives, it is found necessary to Study both the market share for the particular product (figure 5) and the lead time over time (figure 6 &7)).

More frequently, the three phases that govern the pattern of demand for any type of product over a long period of time are; the growth, maturity and decline phase. In this study, the evolution of demand as revealed by empirical data shows increase trend with some fluctuations (growth). In reality, when the fluctuations in demand are high, a reasonable amount of safety stocks are required to avoid stock-out, and as result, holding costs are increased. This issue indicates that controlling the demand function should be one of the principal concern under a backordered case because it may directly affects the inventory level. Another important parameter to control is lead time function. Consequently, controlling the impact of demand and maintenance program on the lead time is another aspect worth spending time on it.

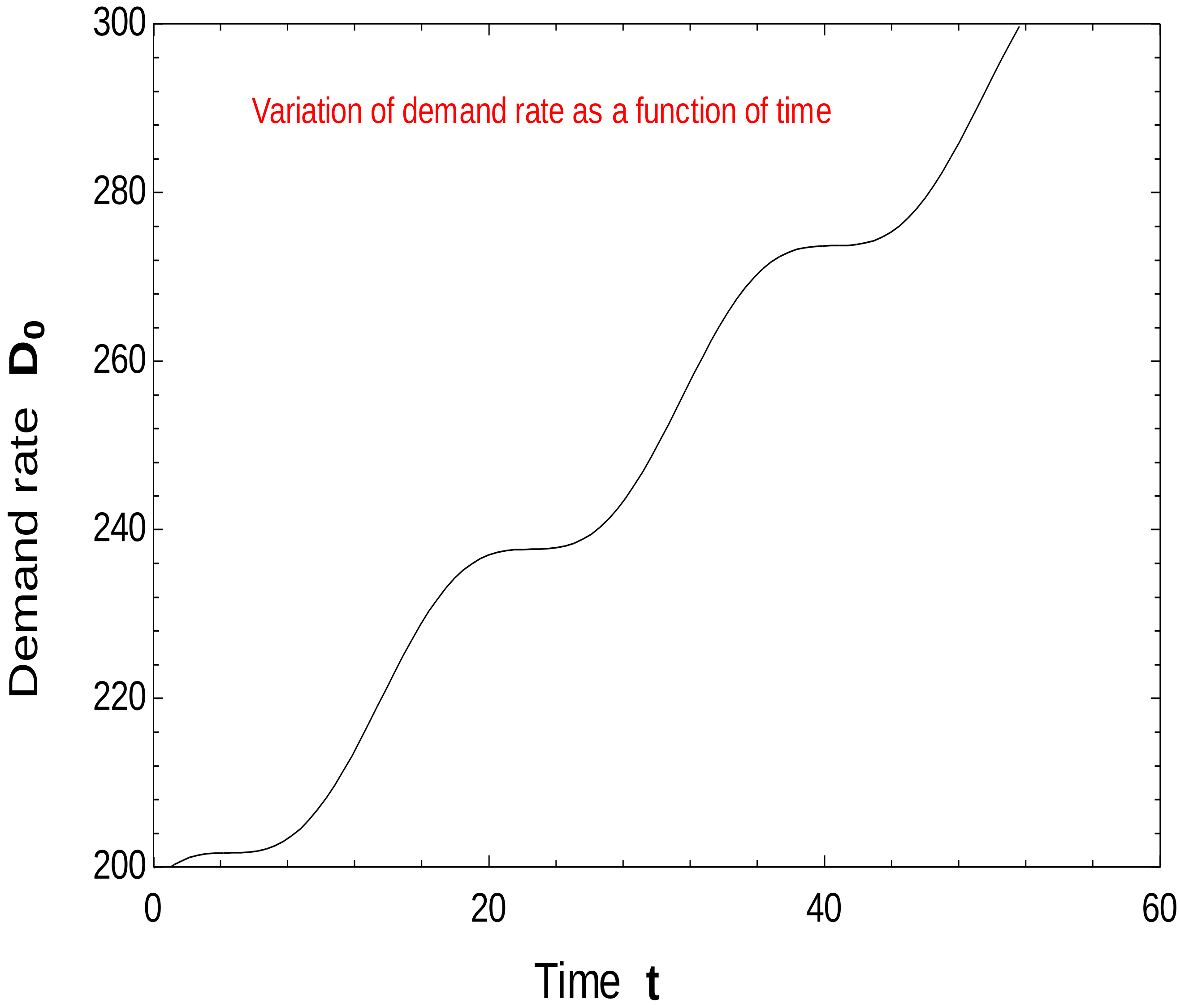


Figure 5: Variation of demand Rate versus time (weeks)

In the next plot, the evolution of the lead time that may be impacted by demand and many other activities is dealt with.

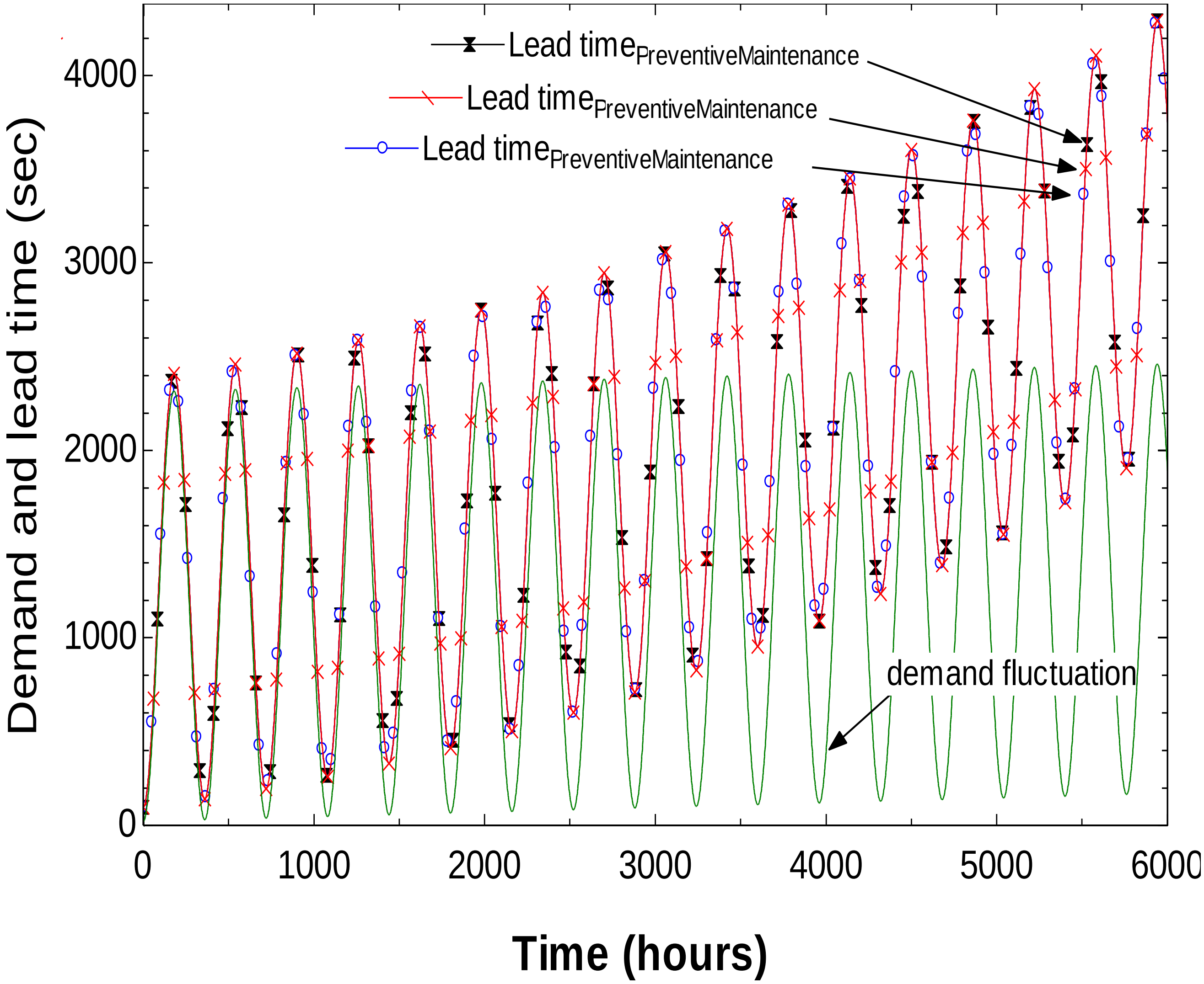


Figure 6: The impact of maintenance program time and demand on the lead time (LT)

It should be observed that the demand volume fluctuates in the same way as lead time, although lead time (LT) increases more rapidly than demand. This rapid increase is because LT is the time that it takes from the receipt of demand-order until when the demand is completely devilvered to the satisfied customer, in which LT is then a function of demand. The deviation between the LT and demand increases as time progresses is shown in figure 6 and figure 7. The higher increase (or deviation) of the LT may be due to the fact that equipments randomly broke down during the protection demand interval, and it took random amount of time maintaining them. The time spent maintaining these equipments then increased the LT and therefore affected the delivery of products to customers. In fact the long run behaviour shows that demand evolves linearly while the TL evolves in a parabolic manner, showing that there was no one-to-one relationship as many be predicted.

The effects of LT and demand level on the inventory level are found in figure 7 and 8. It should be recalled or mentioned here that the inventory trend or inventory level depends on whether inventories are piled up or shipped out. From shipment operation angle, it is observed that a linear increase in demand results in a parabolic decrease in the inventory level. It is obvious that an increase in demand should bring about a decrease in Inventory level for shipment operations, but the interesting revelation is that this decrease in inventory level is not linear as that of demand. The long run behaviour of the shipment operation indicates a point whereby there is rapid increase in inventory level as demand increases before another decay with further demand increase. This abrupt increase is a subject of further investigations which may be attributed to management response to increase in demand.

For inventory pile-up, it is seen that as demand increases linearly, the inventory level increase in a parabolic manner, but this time with the axis of symmetry being the time axis. The abrupt drop in demand is also observed under this scenario, explained to be due to market saturation with fluctuation.

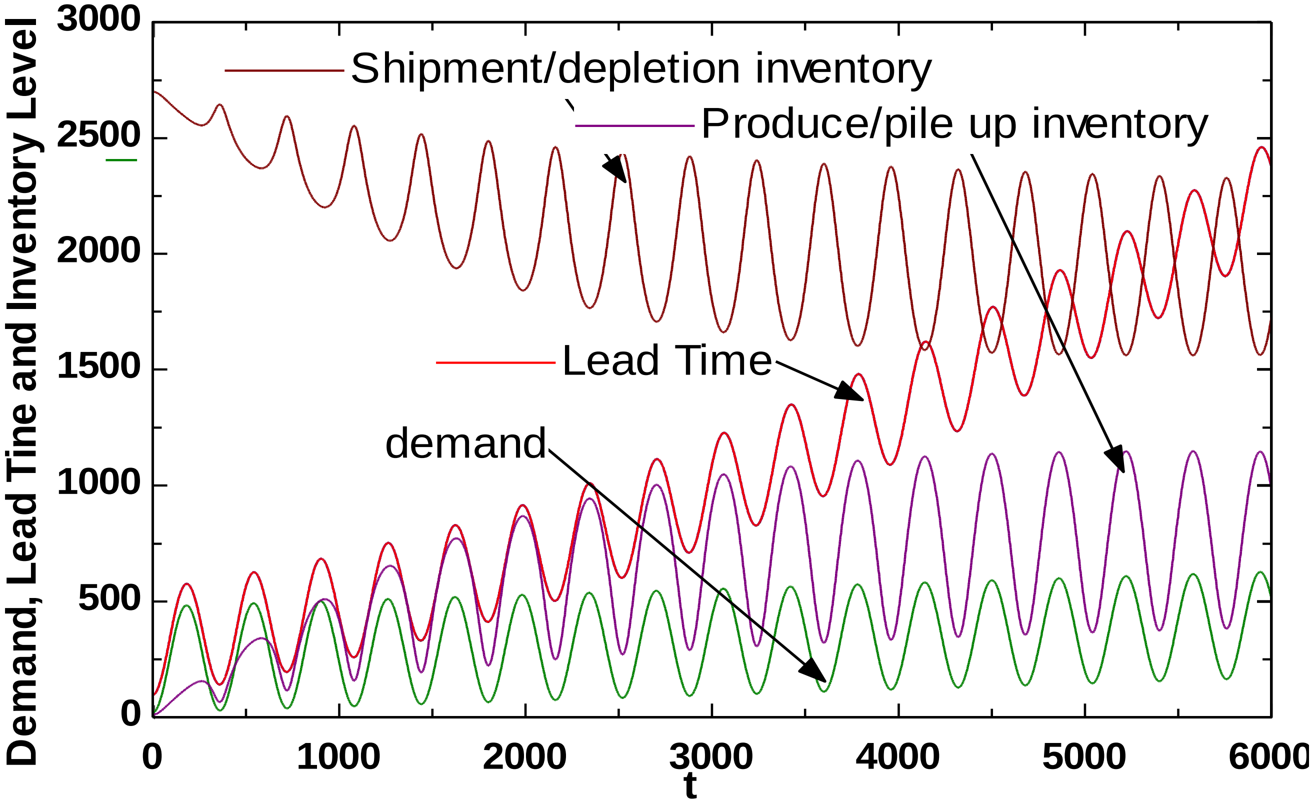


Figure 7 The impact of time on lead time, demand and inventory level

One can then see from figure 7 that the relationships between demand level, maintenance time and inventory level are not one-to-one. Thus, the use of the models proposed here might be beneficiary. From observing figure 8 and 9, more information related to the inventory level and the cost of implementing a continuous *(r, Q)* inventory review system can be gained.

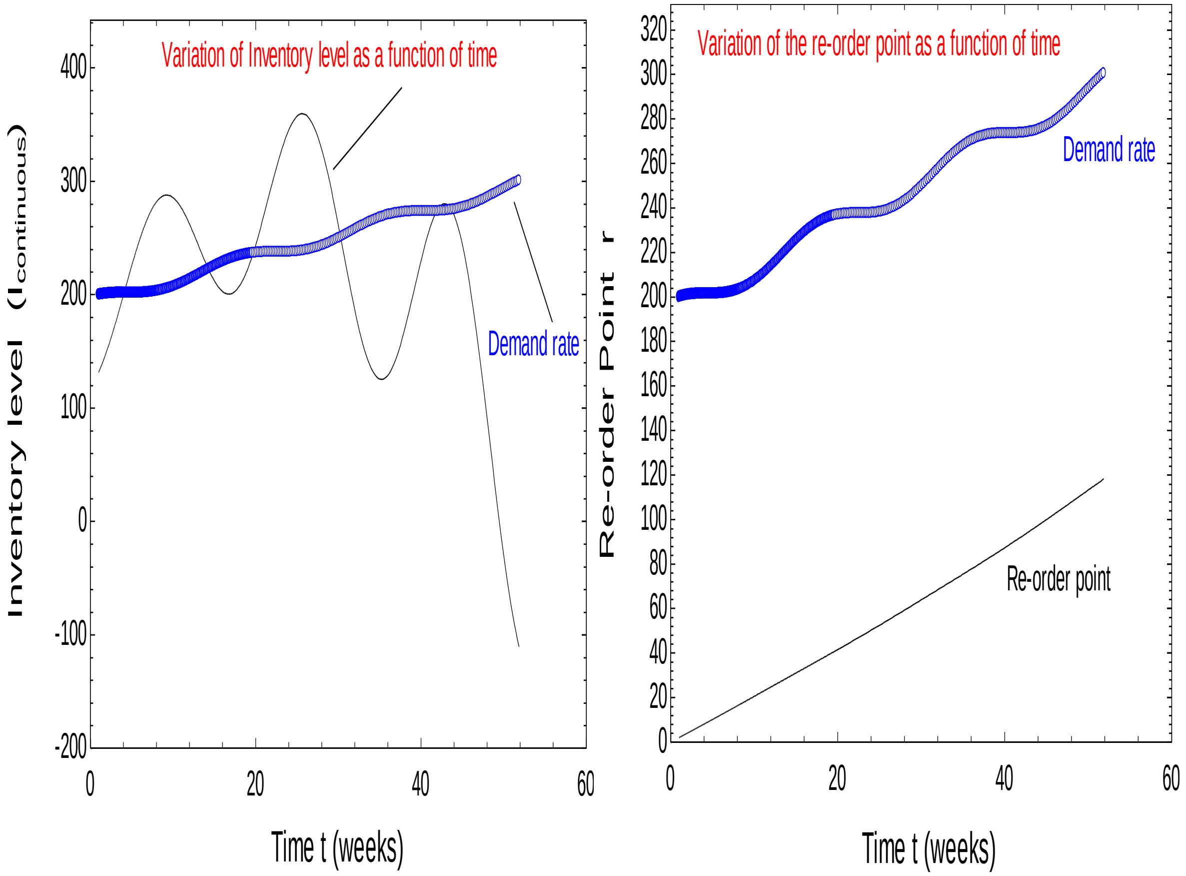
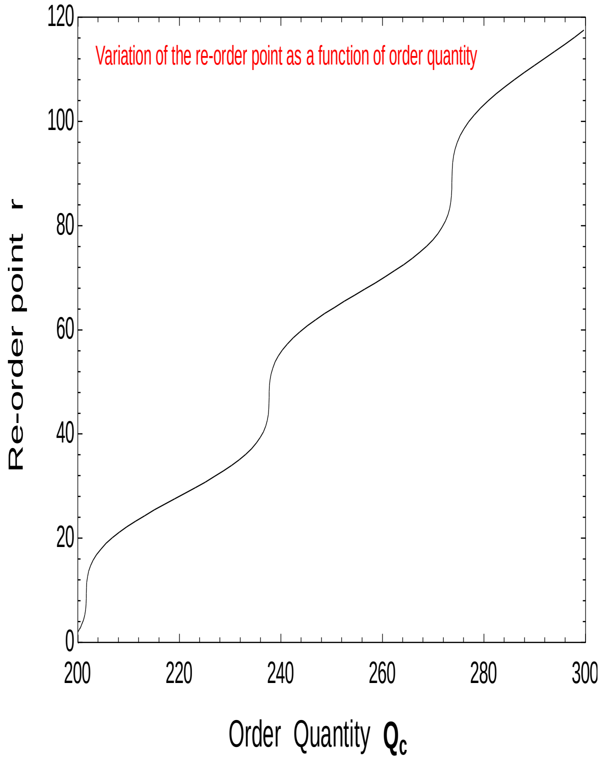
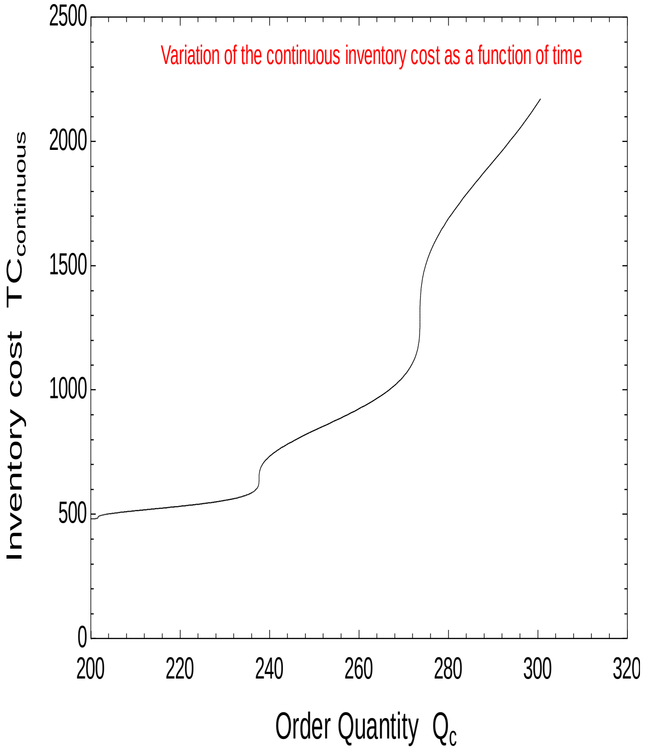


Figure 8: Variation of the continuous inventory level and re-order point as a function of time t (in weeks)

It should be observed that the evolution of the inventory level (figure 8) does not follow the path of demand. An increase in demand is followed by a decrease of the inventory level for the case study company. The evolution of the inventory level presents some fluctuations as time goes by. The decrease of inventory showing negative portrays a high level of stock-out events. This finding has often been mentioned (Anderson, 2008:583). Which is another proof that the proposed models are valid and may be beneficiary. This implies that the increasing demand really has an impact on the inventory level. From a practical point of view, this finding make sense because although the inventory level is initially higher than demand within a certain interval, this inventory path tends to decrease and can be reversed as time goes by. A logical explanation is that the rapid increase of demand may cause the items to be rapidly depleted from the on hand inventory which in turn may lead to stock-out situation or high backorder cost. Hence the need to keep a higher level of inventory in store at the beginning of operation because.

Based on the literatures and discussion with inventory managers at the case study company, two decisions variables are believed to impact the cost of implementing a continuous *(r, Q)* inventory review system. These decision variables include the re-order point and the order quantity. Figures 9 shows the pattern of theses decision variables.





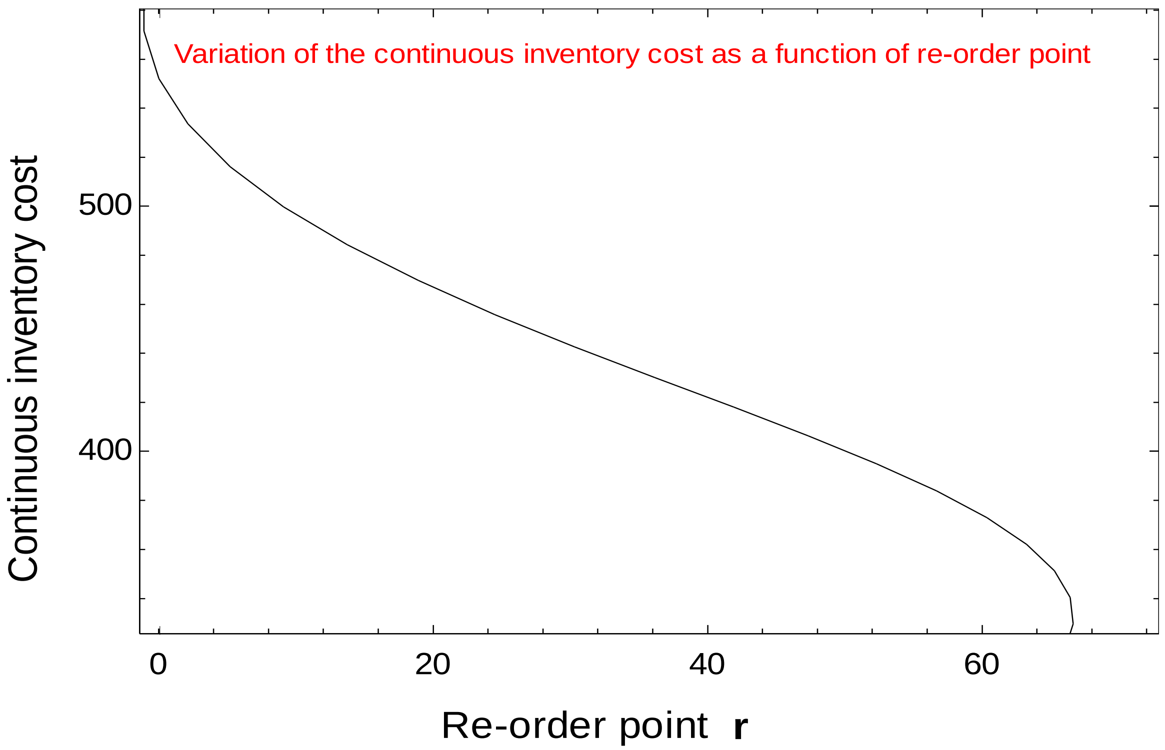


Figure 9: Variation of the re-order point versus order quantity (or lot-size) and re-order point versus cost

It is revealed by figure 9 that there is need to increase reorder point *r* as the order quantity *Q* increases as well with increasing demand. One logical explanation is that, overloaded machine may be an important cause of downtime. Demand that may arise during machine downtime is a situation that should not be neglected because it can lead to high stock-out event. To avoid stock-out events, under such circumstances, the re-order point should be adjusted accordingly. The immediate consequence of this effect is that an increase of the re-order point can contribute to bring the inventory level up which in turn may lead to higher holding cost. Failure to adjust the re-order point without considering the real pattern of demand is therefore risky. Such results may compel one to look for room of improvement. Consequently, the path toward an optimal continuous *(r, Q)* inventory review system are suggested. This situation warrant further examination which is discussed in the next paragraph.

It is also observed that the continuous *(r,Q)* inventory review system tend to produce some peaks in inventory level (figure 9) that can be explained as follow: At arrival of demand, the inventory held increases steadily due to the order quantity. It is further noticed that the continuous *(r, Q)* inventory system cost follows the demand pattern. To be more specific, the inventory cost rapidly increases with fluctuation as the quantity of items needed to cover the expected demand goes beyond a specific limit. The inventory cost also tends to decrease as the re-order point increases. The final observation is the rise in cost after the peaks. These peaks may be due to inventory holding cost, and sometimes the backorder cost. That is the reason why the effect of the backorder is reproduced by the sub-peaks that increase as a function of time.

From a practical point of view, one can say that the company cannot take the risk of decreasing the re-order point as demand increases because it may lead to higher inventory costs.

4.2 Simulation outputs for Periodic *(R, S)* inventory review system

It was also found important to evaluate the company annual total cost and stock out probability of implementing a periodic *(R, S)* inventory review model in an uncertain supply chain. Figure (10), also reveals the path of demand rate, periodic inventory cost and service level.

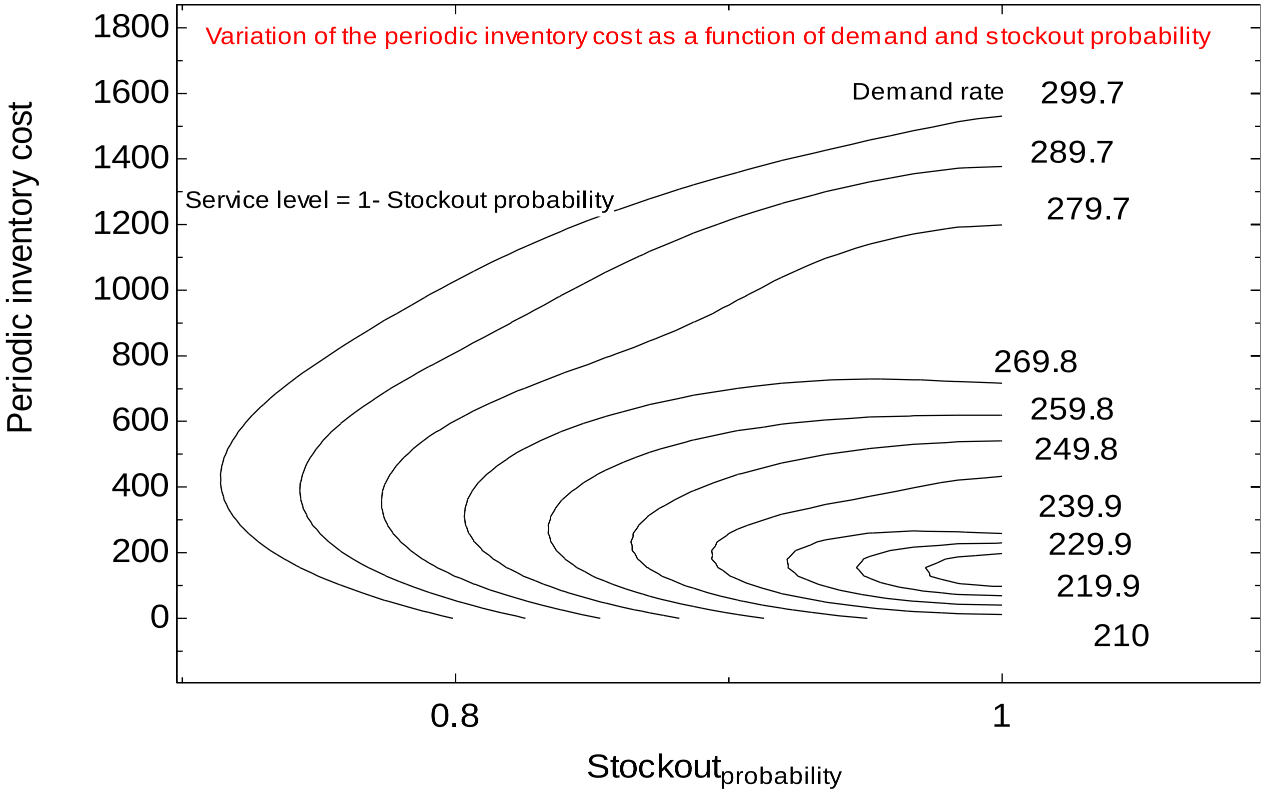


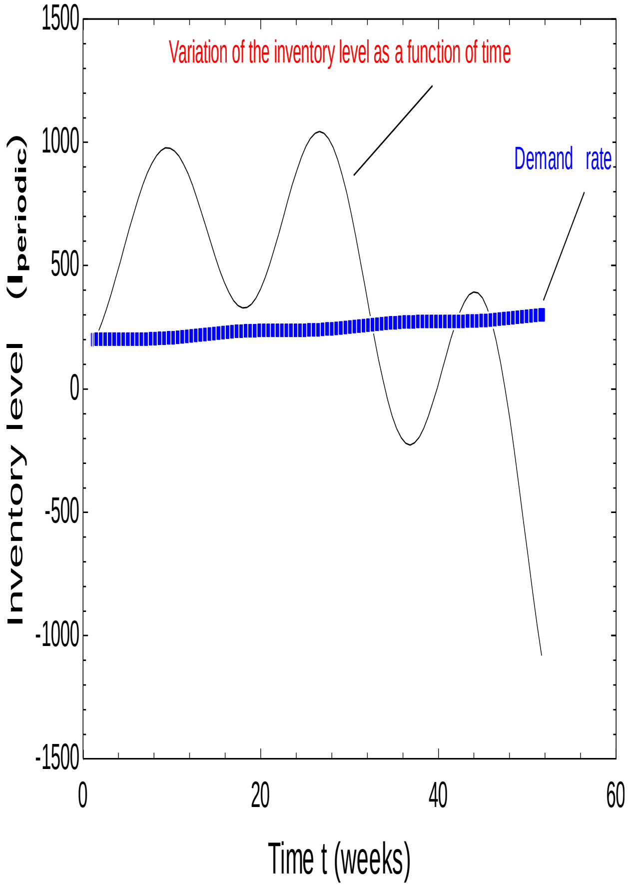
Figure 10: Trend of demand, stock out probability and periodic *(R,S)*inventory cost in a stochastic supply chain

As it can be seen from figure 4, the results observed under a periodic *(R, S)* inventory review system are found to be almost the same as the one presented under a continuous *(r, Q)* inventory review system. This may be due to the fact that managers requested to study the proposed models under the same circumstances. What this finding also portray about the trend of the inventory cost, stock out probability when demand varies is that implementing a periodic *(R, S)* inventory review system will incur higher cost over time. The plots displayed in the next sections are found necessary because of their ability to further help in dealing with the other specific objectives such as to:

- Measure the effect of changes (review interval and order quantity) on the cost of implementing a flexible periodic *(R, S)* inventory review system
- Study critical paths for optimal output

- Establish the relationship between changing protection demand interval, order quantity and the annual total cost change of inventories.

Other results obtained after implementing a periodic *(R, S)* inventory model in a stochastic supply chain are presented in figure 11 and 12.



Variation of the inventory level as a function of time
Demand rate
Inventory level ($I_{periodic}$)
1500
1000
500
0
-500
-1000
-1500
0
20
40
60
Time t (weeks)

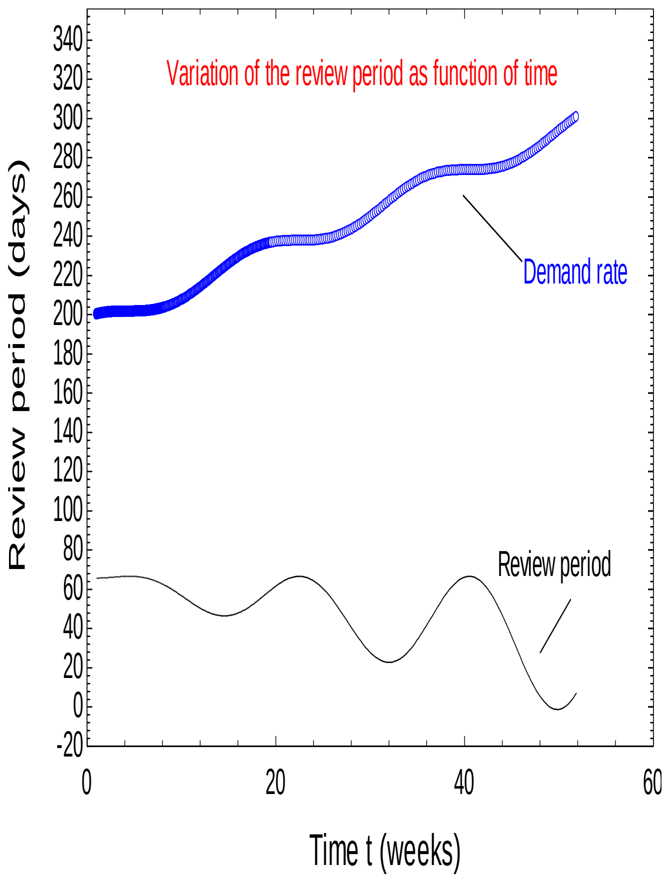
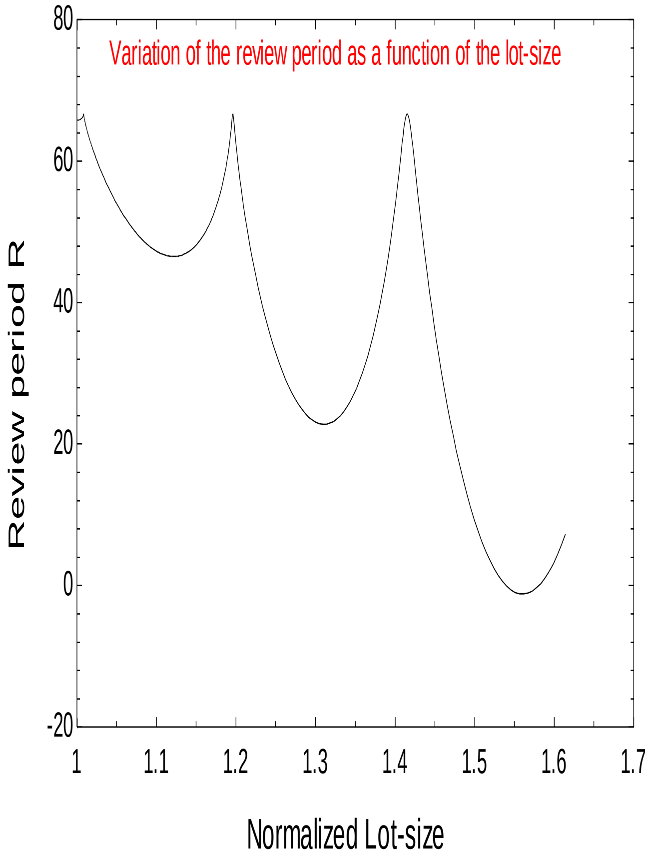
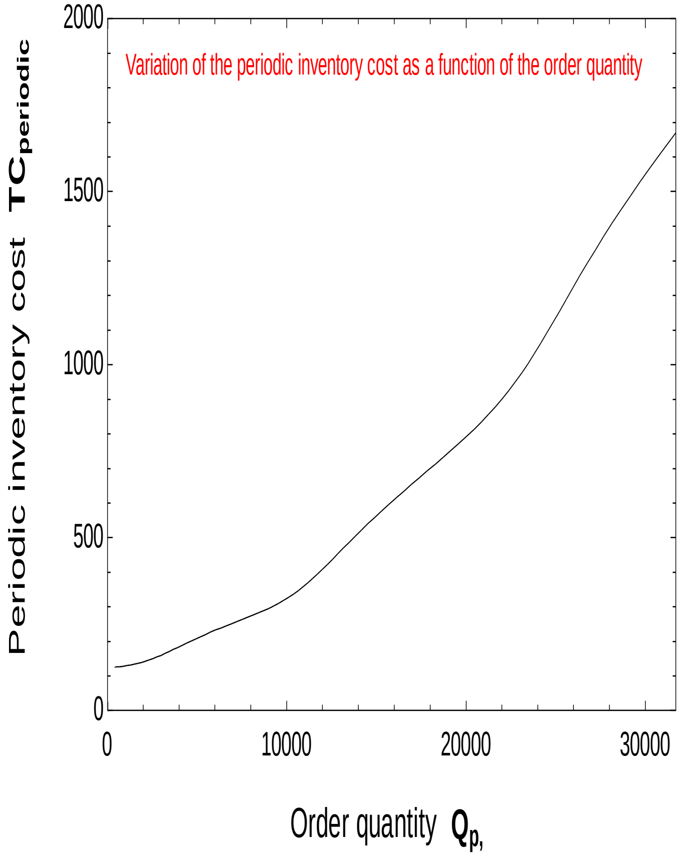


Figure 11: Trend of demand rate, periodic inventory level and review period as function of time

As expected, figure 11 revealed that an increase in demand results in increase in the inventory level, which is followed by a noticeable decrease. The decrease of inventory showing negative portrays a high level of backorder that may occur. This finding has been often mentioned in the literature and other research (whitin, 1976:56). One logical explanation is that the inventory held tend to be affected by orders that may occur before the long protection demand interval (*Review period +Lead time*). The immediate consequence of this effect is that longer review period can also result in higher depletion of inventory from the system which in turn may lead to higher backorder cost. One solution for this problem is to keep enough inventory in store. Unfortunately, keeping enough inventory in store may lead to high costs. A trade-of between demand and the on hand inventory is then required. How the company should deal with the mentioned level of inventory in the system is another issue that need careful attention. It can be useful to reduce the review period simultaneously with slight increase in the order quantity/lot-size (see figure 12).


Variation of the review period as a function of the lot-size
Review period R
Normalized Lot-size
80
60
40
20
0
-20
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7


Periodic inventory cost $TC_{periodic}$
Variation of the periodic inventory cost as a function of the order quantity
2000
1500
1000
500
0
0
10000
20000
30000
Order quantity $Q_{p,}$

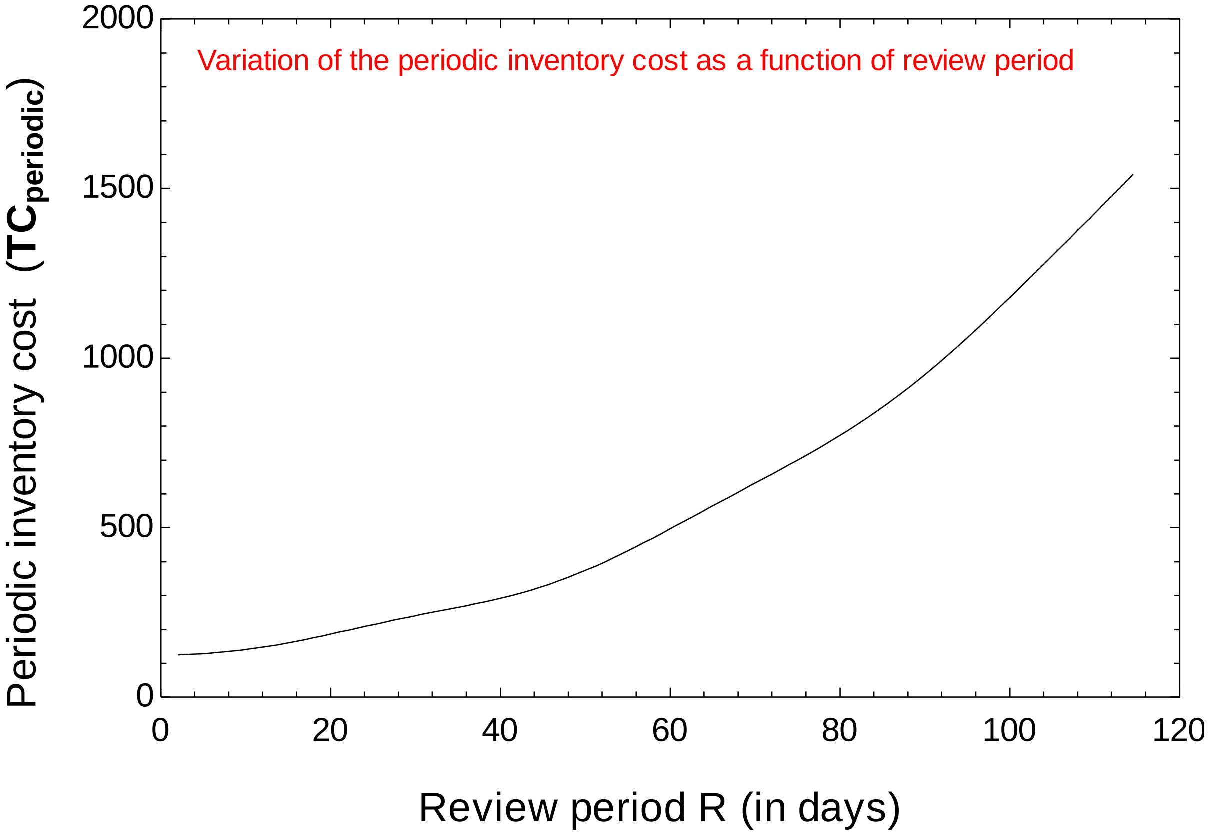


Figure 12: Variation of the review period versus normalized lot-size (order quantity) and inventory cost

Versus order quantity

From observing figure 12, one can notice a decrease in the review period as the lot-size (Truck load factor) increases. What is seen here indicates that while demand may increase, a decrease in the review period simultaneously with an increase in order quantity is required. for optimal cost. Failure to effectively adjust the review period and order quantity/lot-size when demand increases shall lead to high stock out events or backorders cost. The appropriate load or amount that can fill trucks used for delivery purpose may be seen as solution. This result is linked to the literature by (Glock, 2012: 38) who stated that lead time may be reduced by increasing producing rate or increasing the lot size. In many practical ways, by reducing the lead time, it is possible to lower the safety stock, reduce the stock-out level and improve the customer service level.

Figure 11and 12 also reveal that longer review period may cause the system to have higher backorder inventory which in turn may lead to higher inventory cost. For instance, when the time between reviews become longer than expected, the order quantity should be higher in order to prevent a stock out situation that may occur. However, the reverse event should be observed when the time between reviews *R* becomes lower than expected. Care should then be taken in determining the optimum size of the order quantity/lot-size which definitively depends on the review period. Thus, company may avoid such situation by keeping the right balance.

4.3 Simulation outputs for Hybrid inventory review system

Recall that this section establishes the effect of implementing simultaneously periodic *(R, S)*, and continuous *(r, Q)* models. Applying the hybrid inventory model yields the most optimal solutions which are well described by the plot of figure 13.

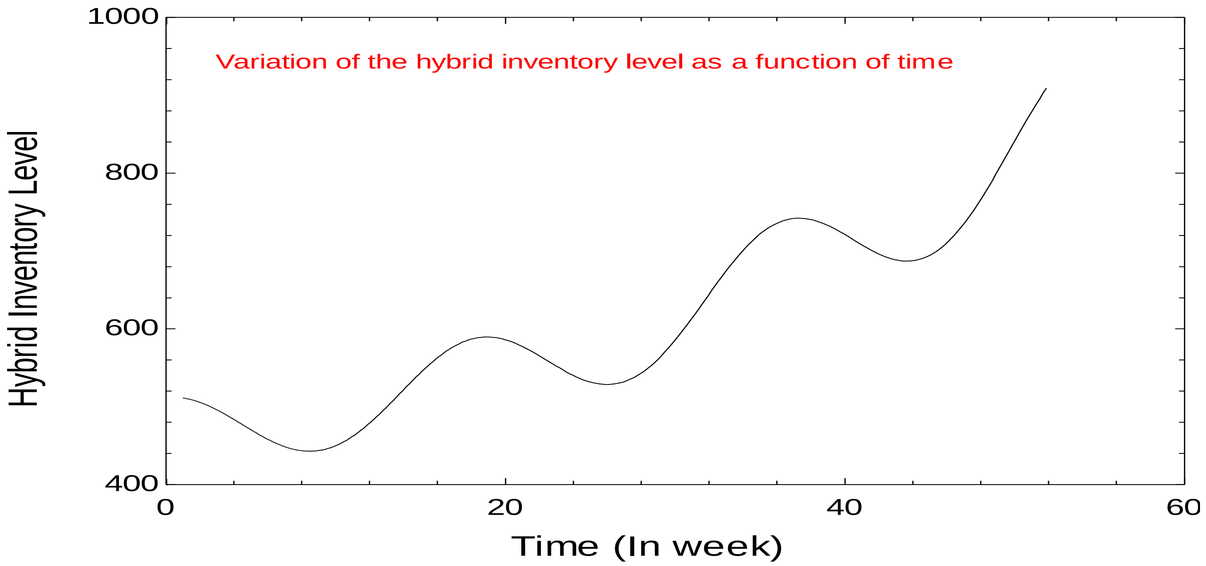


Figure 13: Variation of the hybrid inventory level versus time

It can be seen from the plot (figure 13) that such hybrid inventory system starts with almost a low level of inventory in store. However, as time goes by the level of inventory at hand rapidly increases because it is forced to follow the path of demand which increases with fluctuation. Although the inventory held under such hybrid system evolves following the path of demand, it does not allow any stock-out situation or backorder cost before the review date. This is an interesting result because the total inventory cost decreases over time. A logical explanation of this finding is that as demand increase, the re-order point increases as well. This implies the rise of the on hand inventory. In such situation, the review period is expected to decrease (see figure 14) so that it agrees with the path of the on hand inventory level. It is easy to understand that if inventory managers are willing to often check the inventory level before the review date, they will effectively replenish the hybrid inventory system and save a significant percentage of ordering cost and backordering cost. This shows that good coordination of replenishment activities may lead to saving.

The hybrid inventory held and the path of demand match with each other as expected. This indicates that the use of such hybrid inventory model can be beneficial for the company because it may lead to considerable inventory cost reduction through the avoidance of stock-out events. Figure 14 further shows the possible benefit of implementing such hybrid inventory review model.

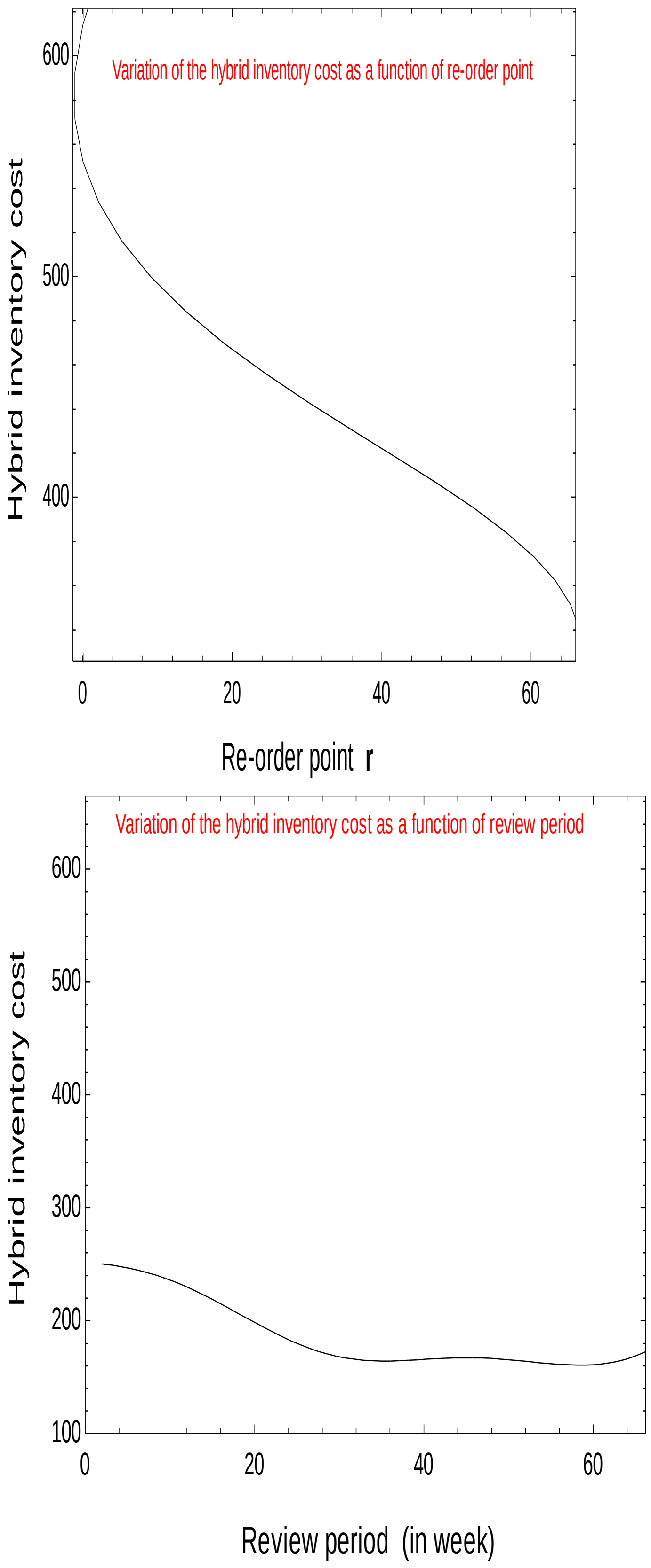


Figure 14: Variation of the hybrid inventory cost versus re-order point and review period

The plot (figure 14) reveals the path of the hybrid system cost versus re-order point and review period. In fact, the maximum hybrid inventory cost is reached when the re-order point is equal to zero (when there is not inventory in store). This simply implies that the hybrid system is out of stock. An event that can only occur at the beginning of operation. The high cost incurred at the beginning of operation may be alluded to stock-out events. As demand increases, the reorder point increases as well which in turn lead to inventory cost being the sum of holding and ordering cost alone. One can clearly see that the total hybrid system cost decreases because stock-out event are totally avoided. The strongest reason for having such result may be due to the fact that stock-out events are frequently well dealt with as demand increases. Although demand increases over time, note that it was observed that there were some situations where customers were willing to wait.

Optimal adjustment on the re-order point may cause the hybrid inventory system to incur lesser on hand and ordering inventory. From a practical point of view, it is correct because having lesser inventory in store while managing demand that increase should be the ideal case scenario to be achieved by every firm. This situation can be effectively managed if customers agree on a certain reasonable delay. Hence, the need for inventory managers to know how to deal with the re-order point under the hybrid inventory system. More insight can also be gained by studying the relationship that exist between the cost of implementing a hybrid inventory system, and the order quantity.

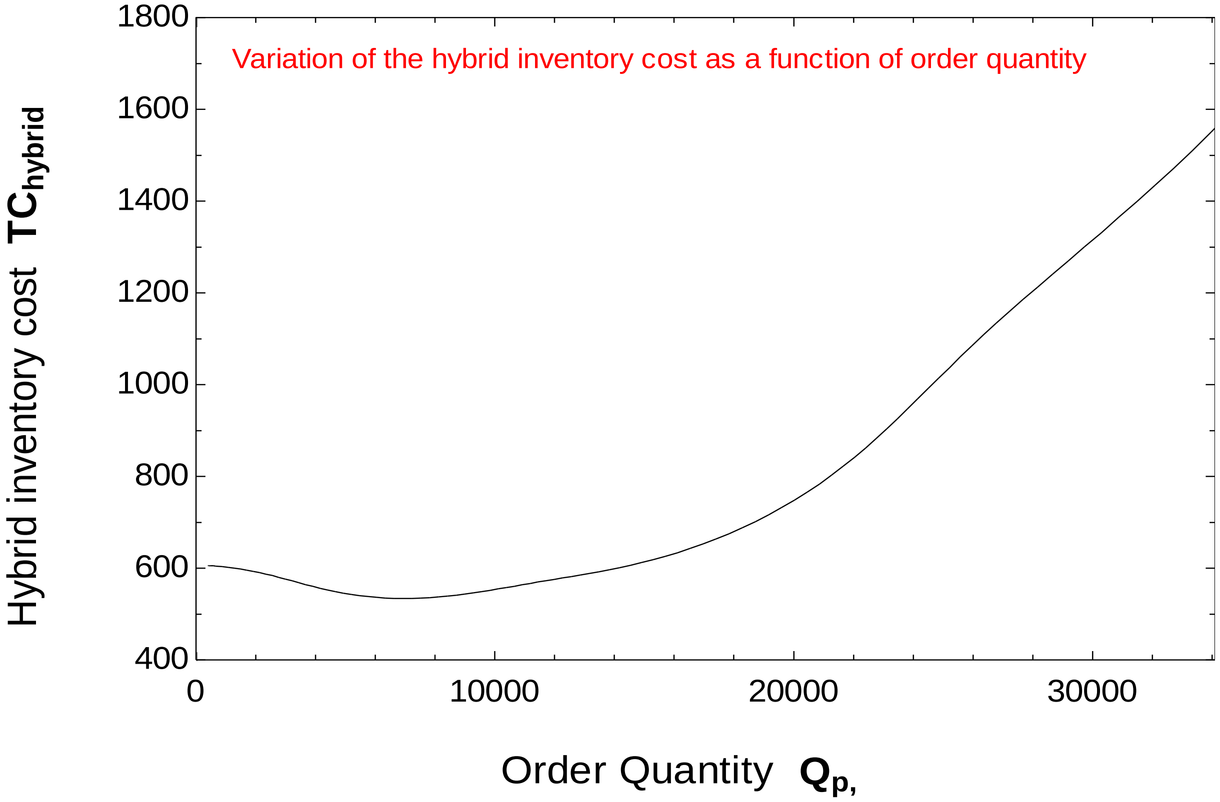


Figure 15: Variation of the hybrid inventory cost versus order quantity

Figure 15 reveals that the hybrid cost tend to be low and almost constant when no order is placed, and it decreases slightly until a critical point is reached as the order quantity slowly increase. Whenever the order quantity is greater than the critical point or optimum point, a parabolic increase of the hybrid inventory cost is observed. Another interesting result is revealed. As the number of replenishment tend to be high, the holding inventory cost ends up being low. The trend is reversed once the critical point is reached. In this case, as the number of replenishment decreases the ordering inventory cost ends up being high (see figure 16).

There are two possibilities involved. First of all, the manager checks the inventory level more frequently and may find himself ordering in small quantity. The logic behind this operation is obviously clear because if one order frequently, the size of the order should automatically decrease. In other word, the uncertainty is well dealt with by ordering frequently. Secondly, the manager reduce the order frequency and raise the order size. This operation may require a larger size of the order quantity which may also lead to the reduction of the stock-out probability and increase of the on hand inventory cost. The immediate consequence of having such knowledge may cause the manager to accurately order and avoid backorder inventory cost. To further understand the impact of the number of replenishment on the inventory cost, it is necessary to analyse figure 16.

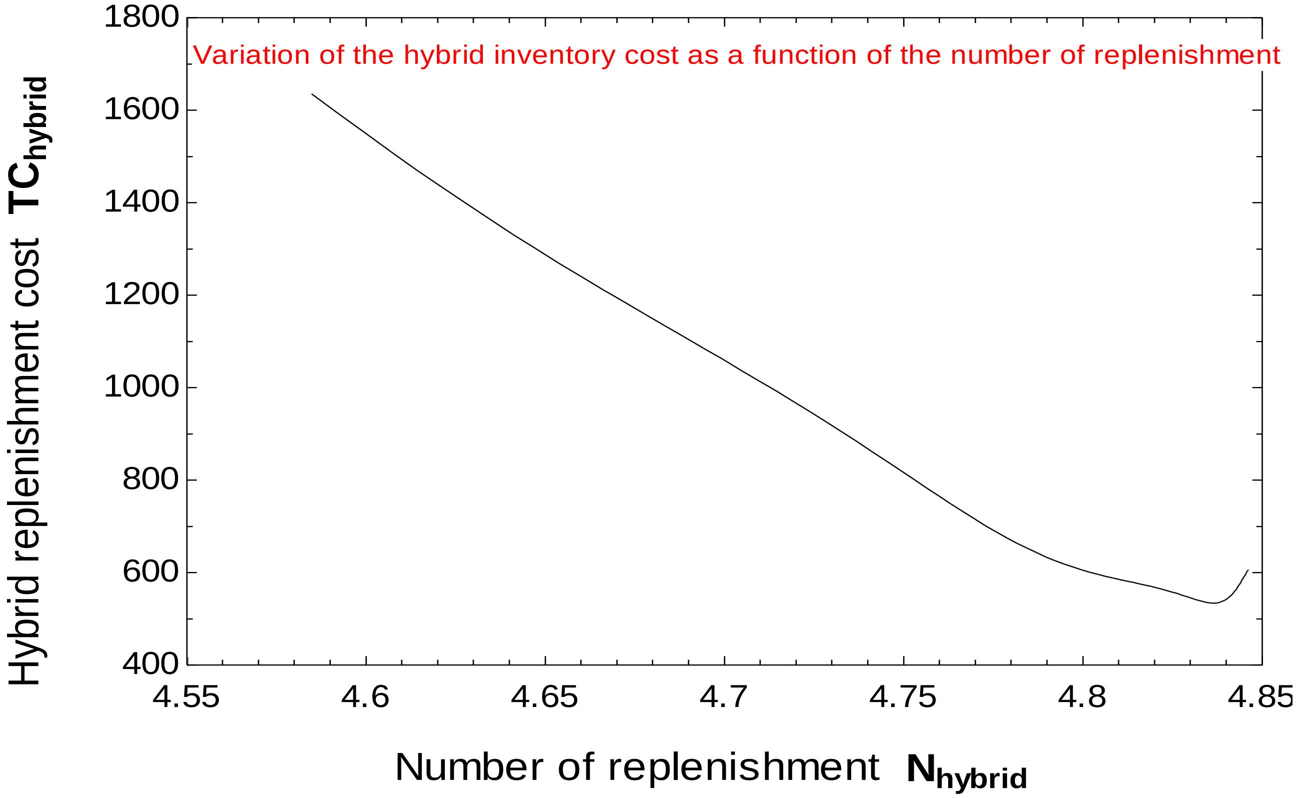


Figure 16: Variation of the hybrid inventory cost versus number of replenishment

Figure 16 reveals the relationship between the number of replenishment and hybrid inventory cost. As the number of replenishment increases, the hybrid inventory cost decreases. So, this path indicates that while the ordering cost increases, the total inventory cost should decreases. This implies a reduction of the stock out probability which lead to low backordering cost. It is obviously clear that a system whose inventory level is effectively checked would automatically lead to low inventory cost when demand increases. The question now is to determine how often the replenishment process may take place in order to satisfy a particular demand. There must be an optimal point for the number of replenishments, which is clearly revealed in figure 16. This shows that high stock-out probability can effectively be dealt with by using a hybrid inventory model. A hybrid inventory model as designed in this thesis can be used because it includes an interaction between the continuous*(r, Q)* and periodic *(R, S)* inventory system and can help the system to meet high performance specification.

4.4 Comparison between Continuous *(r, Q)* and periodic *(R, S)* system

From the previous result obtained, a comparison of the cost of implementing continuous *(r, Q)* and periodic *(R, S)* inventory review model over time is presented by the plots (see figure 17).

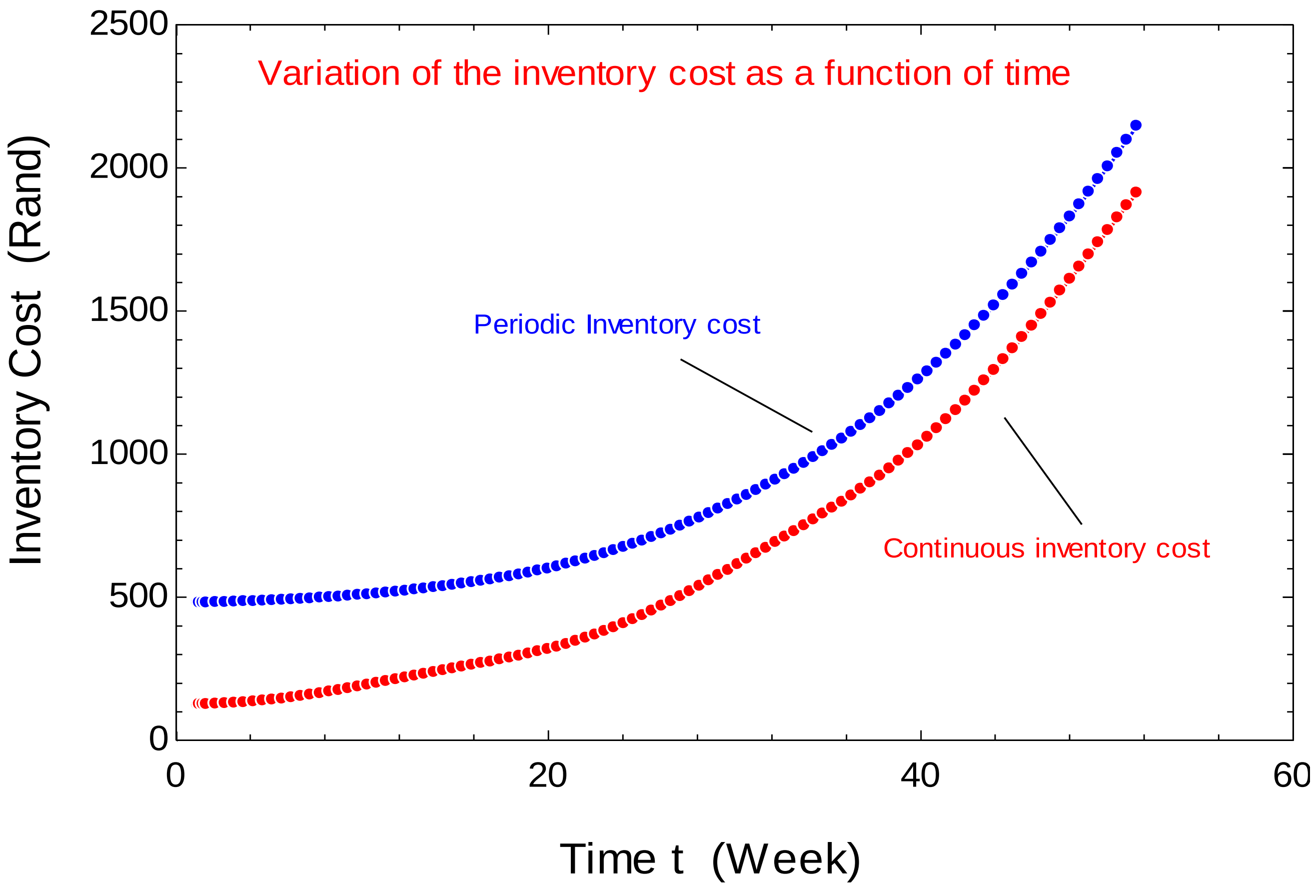


Figure 17: Variation of the periodic *(R, S)* and continuous *(r, Q)* inventory costs versus time

It is found that implementing a continuous*(r, Q)* inventory system tend to incur lesser cost than a periodic *(R, S)* inventory system over a period of 52 weeks (year). Although both review system showed a noticeable difference at the beginning of operation, one is also to notice a reduction of that difference as time goes by. The strongest reason for having a noticeable difference in the result as described by figure 17, may be due to the fact that the protection demand interval under the periodic *(R, S)* inventory system has a greater value than that of continuous *(r, Q)* review system. Another reason for having such difference in the result is that both inventory review system operate differently. The final reason is found in the use of a different problem solving approach. It should finally be observed that a graphical comparison (the usage of graphs) may appears more transparent and easy interpretable when compared with data reported under different method (Merkuyeva, 2010:187).

It should also be observed that a continuous*(r, Q)* inventory system tend to incur lower cost at beginning of operation while the reverse is observed under a periodic *(R, S)* inventory system. The closing up of the gap between the two system may be due to frequent adjustment of the re-order point and review period when demand increase with fluctuation. In fact, change in the re-order point which is mainly initiated by demand that affect the order quantity which in turn affect the ordering process and the amount of inventory at hand. Hence the need to effectively manage the re-order point and review period because they can either highly lead to higher holding cost or ordering cost or both. In the same order of idea, the result suggests that a continuous*(r, Q)* inventory system be used, especially when demand has an increasing path with some fluctuation. However, periodic *(R, S)* inventory system may be more preferable in practice since it is easier to manage and control.

From a practical point of view, the result implies that the time spent maintaining some equipment after they break down may be long and stochastic. Therefore, it may be more expensive to use a periodic *(R, S)* inventory system than a continuous*(r, Q)* inventory system when the protection interval demand varies. One logical explanation is that the protection demand interval seem to be longer under a periodic *(R, S)* inventory review system than a continuous*(r, Q)* inventory system. This indicates that more attention should be devoted to manufacturing and transportation times' reduction if one is to reduce the protection demand interval under both inventory review policies. Thus, the models proposed are found being beneficial. To further examine the effectiveness of using continuous *(r, Q)* and periodic *(R, S)* inventory review models, it is found necessary to see to what extend such hybrid models that result from their combination can help one to meet high performance specification.

4.5 Comparison between Continuous *(r, Q)*, periodic *(R, S)* and hybrid system

The next plot seeks to establish the relationship that exists between the hybrid inventory system cost, the cost of periodic *(R, S)* and continuous *(r, q)* inventory systems.

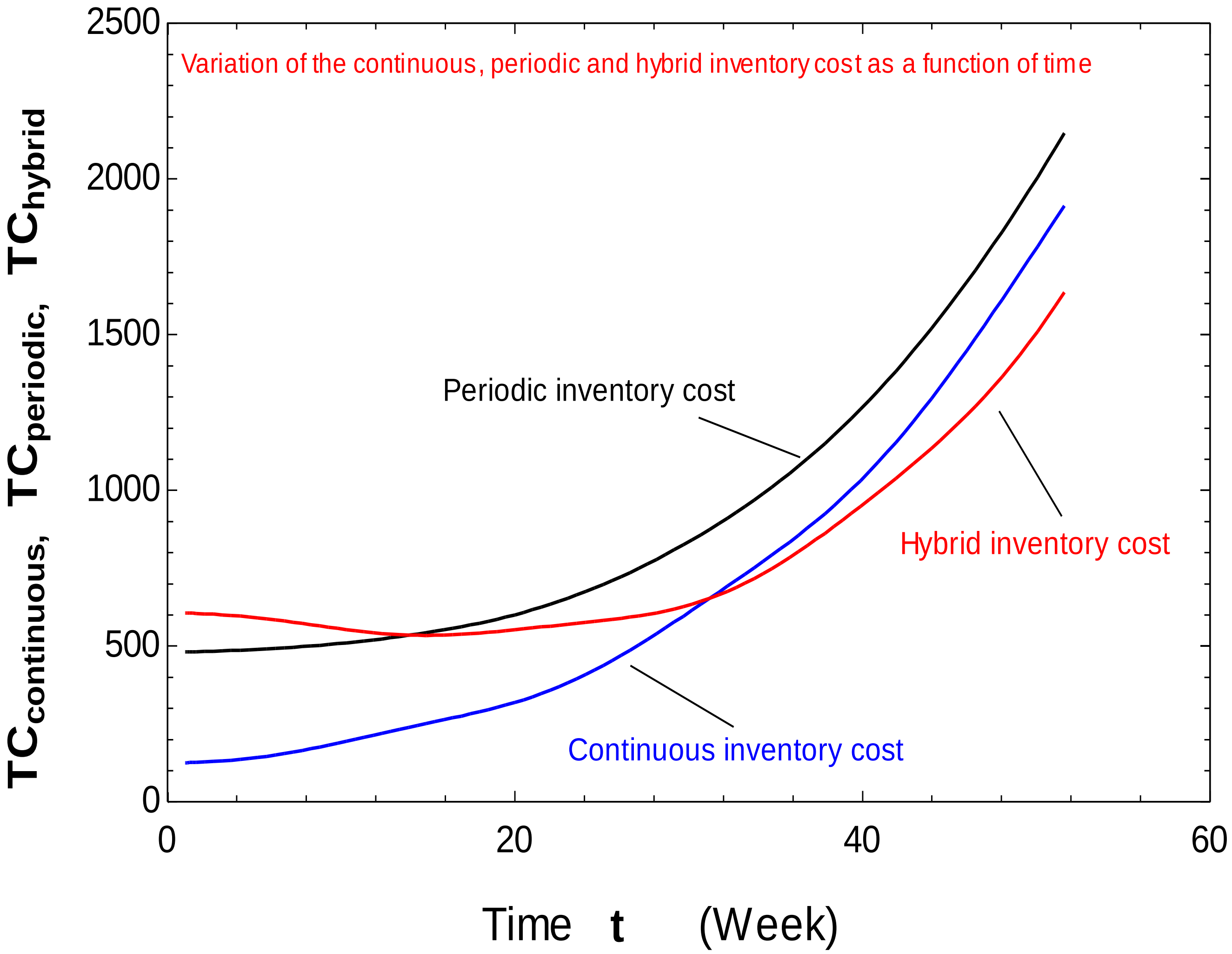


Figure 18: Variation of the continuous, periodic and hybrid inventory cost

To sum up, in this section the results of the proposed flexible continuous*(r, Q)*, periodic *(R, S)* and hybrid inventory system are presented in the same figure 18. The result reveals that it is natural to use continuous*(r, Q)* and periodic *(R, S)* inventory system individually in order to help the system to meet high performance specification. However, some issues are observed:

- Implementing a periodic *(R, S)* inventory system incur lesser cost at the beginning of operation than a hybrid inventory system
- Implementing a continuous*(r, Q)* inventory system incur lesser cost at the beginning of operation than a periodic *(R, S)* inventory system
- Implementing a continuous*(r, Q)* inventory system always incur lesser cost throughout operation than a periodic *(R, S)* inventory system, but this difference tend to decrease as time goes by.

One can conclude that the difference found in the length of the protection interval demand may be the main reason. From a practical perspective, the length of the protection interval demand under a periodic *(R, S)* inventory system is designed to be longer than the one under continuous*(r, Q)* inventory system. Operationally speaking, this can create a huge difference when an increasing demand with fluctuation arise. For instance, under a periodic *(R, S)* inventory system, the longer protection interval which is composed of the review period plus lead time demand may compel one to raise the size of the initial inventory so that the system can effectively deals with the path of demand. Knowing the impact of the review period plus lead time on the inventory cost, one can say with confidence that the high inventory cost observed at the beginning of operation is justified. However, under a continuous *(r, Q)* inventory system, because the protection demand interval is composed of the lead time alone, its impact on the inventory cost observed at the beginning of operation is lesser. This result agree (match) with the practical reality.

The result also reveals that it is not always desirable but also natural to use a hybrid inventory system (that results from a combination of continuous*(r, Q)* and periodic *(R, S)* inventory system) in order to help the system to meet high performance specification. However, the hybrid inventory is the only one that can yield lesser cost over time. This result also indicates that combining the feature of both continuous*(r, Q)*, and periodic *(R, S)* inventory system is an approach that deserve to be considered because it may lead to low cost. Note that the strongest reason for combining the feature of both continuous*(r, Q)*, and periodic *(R, S)* inventory system is to come-up with a hybrid system that effectively deal with backorder inventory and the replenishment process. By doing so, one can effectively reduce the total inventory cost through reduction or elimination of the cost related to backorder inventory and ordering cost. The path of the hybrid inventory held and number of replenishment as described in figure 13 and 16 respectively can serve as evidence to support this argument. It is easy to conclude that the hybrid system cost composed of the ordering and holding cost should be the one to be dealt with. For all these reasons, it is found more beneficial to use the hybrid inventory model in order to address the inventory problem.

Chapter 5 Conclusion and recommendation

In this thesis, a case study of a manufacturing company around the Vaal region is used to illustrate the benefits of simulation models, in other word, this dissertation shows how simulation optimization can be used to investigate the effects of the stochastic demand and lead time on the cost of implementing a continuous *(r, Q)*, periodic *(R, S)* and hybrid inventory review models. In addition, one can explore the effect of change in many important factors such as variation in re-order point and review period on the inventory cost.

From the simulation outputs, it is also observed that as the demand increases, the reorder point for continuous *(r, Q)* review system should increase as well. Such event should compel the inventory manager to raise up the size of the order quantity accordingly. This emphasizes the importance of choosing the re-order point that match the order quantity so as to have optimised cost. Further, it is revealed that the review period should slowly decreases as the order quantity increases under a periodic *(R, S)* inventory review system for optimal cost. It was found that the interaction between lead time and review period is of great importance. It means that when the protection demand interval is longer than expected, adjusting the review period can lead to total inventory cost. This also emphasizes the importance of choosing the review period that match the lead time because it may affect the order quantity.

As can be observed from the previous plots, one can say that a periodic *(R, S)* inventory review system suffer more from demand and lead time variability than a continuous *(r, Q)* inventory review system under a stochastic environment. Thus, the use of the a periodic *(R,S)* model proposed here might be less beneficiary as it indicates higher costs in comparison to a continuous *(r, Q)* inventory review system. It is finally revealed that a hybrid inventory system that results from a combination of continuous*(r, Q)* and periodic *(R, S)* inventory system is more cost effective.

As one can see from reading the information presented, implementing continuous *(r, Q)*, periodic *(R,S)*and hybrid inventory review models in most companies-supply chains system lead to considerable cost increase. These increase in cost may affect the system flexibility which in turn may affects the level of service required to satisfy customers. The high costs due to the uncertainty may be as a result of the fact that Lead Time and Demand that are the main input parameters of every inventory system cost. Most of the time, they are not designed to be fully utilized under different and uncertain conditions such as seasonality, poor manufacturing, poor supplies and delivery performance, etc. Thus, the results of the simulation can be used to gain better understanding of different inventory policies and help to select the configuration that suit the company main objective.

Although the proposed models help inventory managers to deal with items having a probabilistic demand and protection demand interval, they are still not complete and many of the latest inventory concepts have to be incorporated in these models. This can surely help as a catalyst for the development of more realistic and pertinent models. Therefore, more attention should be devoted to formulating accurate models for Lead time and demand that incorporate uncertainty under specific circumstances.

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APPENDIX

Appendix 1 Peer-reviewed research output published

This study has already produced the following peer-reviewed research output published by the digital data base of the institute of Industrial Engineers.

- A conference paper for SAII25 Proceeding July 2013, entitled "Optimizing inventory ordering policies with random lead time"
- A conference paper for CIE44 & IMSS'14 Proceeding, 14-16 October 2014, Istanbul/Turkey entitled "Optimizing the cost of implementing a periodic *(R, S)* and continuous *(r, Q)* inventory review model with stochastic lead time and demand**"**

Appendix 2 (Demand and lead time data collected)

Company In The Vaal Region
Demand data recorded in 2012-2013

Week	**Demand**	**Lead Time(in hours)**
1	200	1
2	207	1
3	216	1
4	219	1
5	208	1
6	201	1
7	190	2
8	215	1
9	197	2
10	202	1
11	207	1
12	212	1
13	205	1
14	222	1
15	240	1
16	230	1
17	205	1
18	250	2
19	236	3
20	242	1
21	248	2
22	250	1
23	238	1
24	228	1
25	227	1
26	221	1
27	227	2
28	234	1
29	247	1
30	271	2

31	273	1
32	259	1
33	283	1
34	275	1
35	277	2
36	288	2
37	251	3
Week	**Demand**	**Lead Time(in hours)**
38	293	2
39	284	1
40	274	4
41	263	2
42	254	4
43	254	1
44	245	2
45	257	1

46	263	3
47	282	6
48	276	2
49	293	3
50	301	1
51	258	3
52	301	1
Week	**Demand**	**Lead Time(in hours)**
53	307	2
54	316	3
55	319	1
56	299	4
57	312	1
58	293	1
59	300	2
60	256	1
61	278	4

62	250	3
63	262	2
64	273	2
65	246	1
66	229	3
67	253	1
68	258	3

Week	**Demand**	**Lead Time(in hours)**
69	239	2
70	241	1
71	359	3
72	259	1
73	300	4
74	288	7
75	299	3
76	315	6
77	327	4

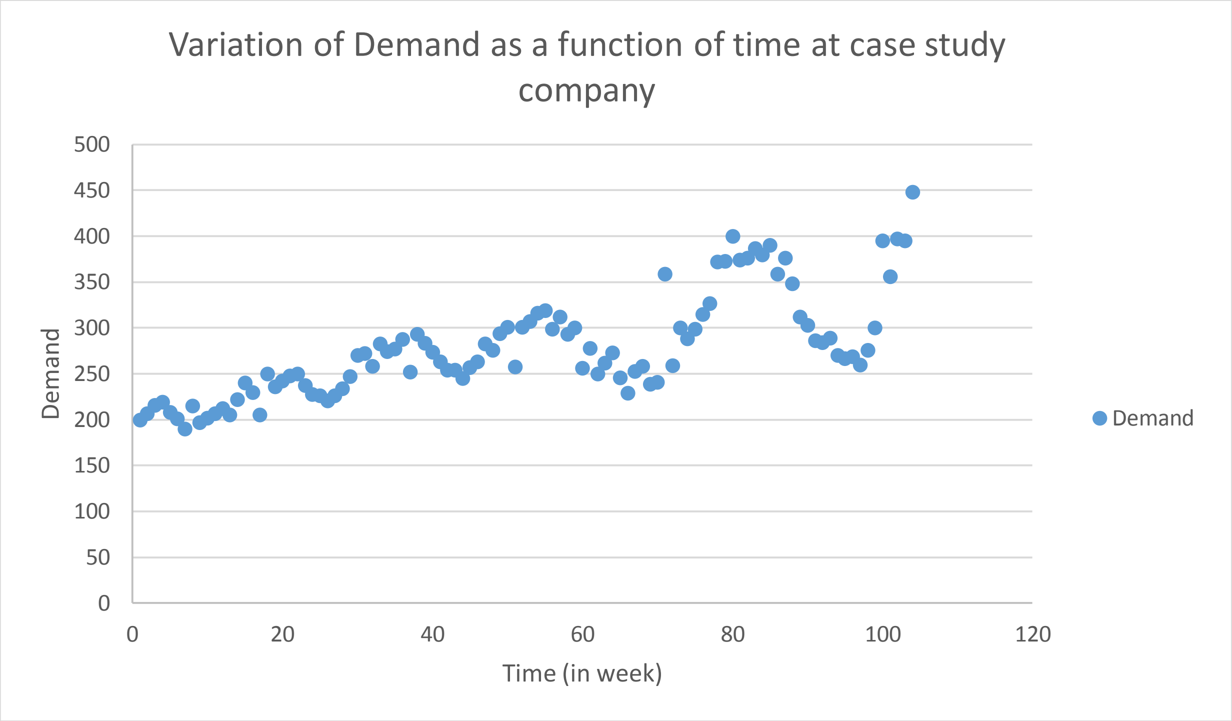
78	372	8
79	373	8
80	400	7
81	374	6
82	376	7
83	387	1
84	380	8

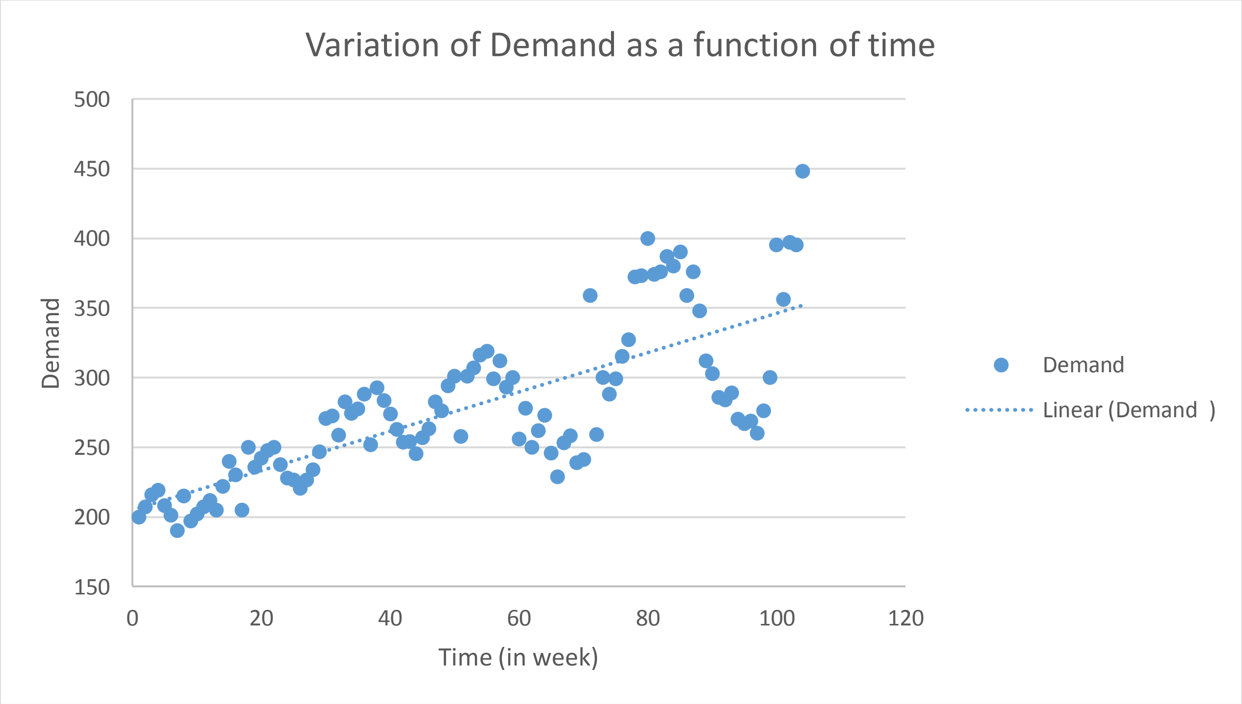
Week	**Demand**	**Lead Time(in hours)**
85	390	9
86	359	7
87	376	12
88	348	6
89	312	22
90	303	2
91	286	1
92	284	2
93	289	5

94	270	8
Week	**Demand**	**Lead Time(in hours)**
95	267	8
96	269	7
97	260	5
98	276	8
99	300	7

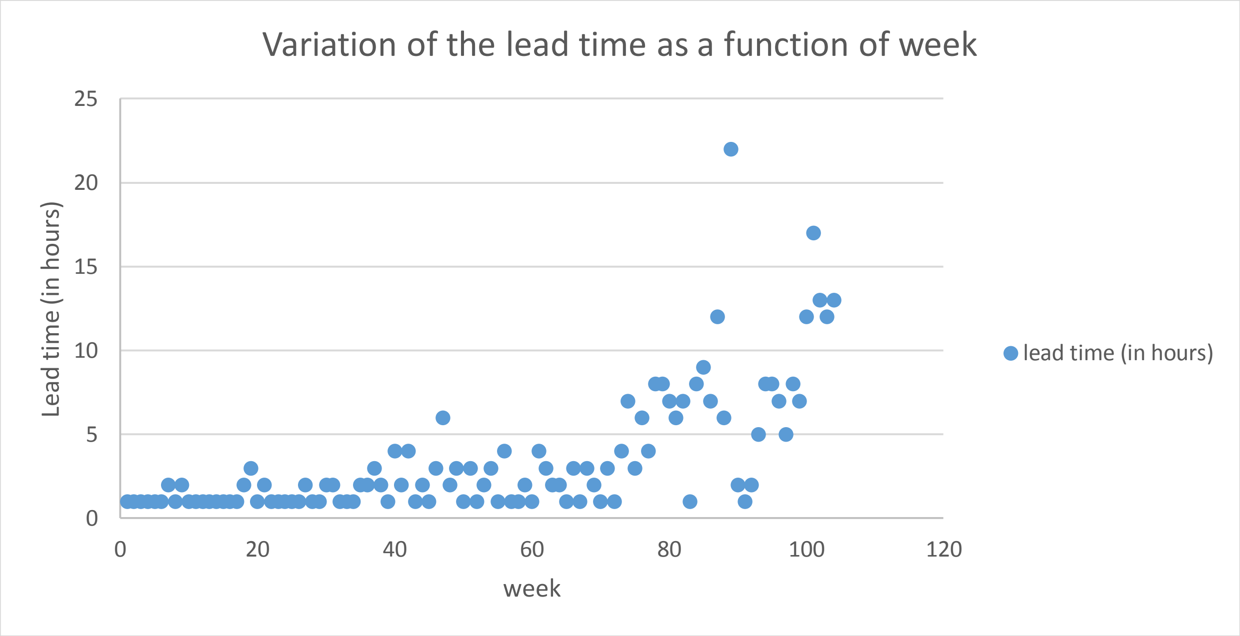
100	395	12
101	356	17
102	397	13
103	395	12
104	448	13

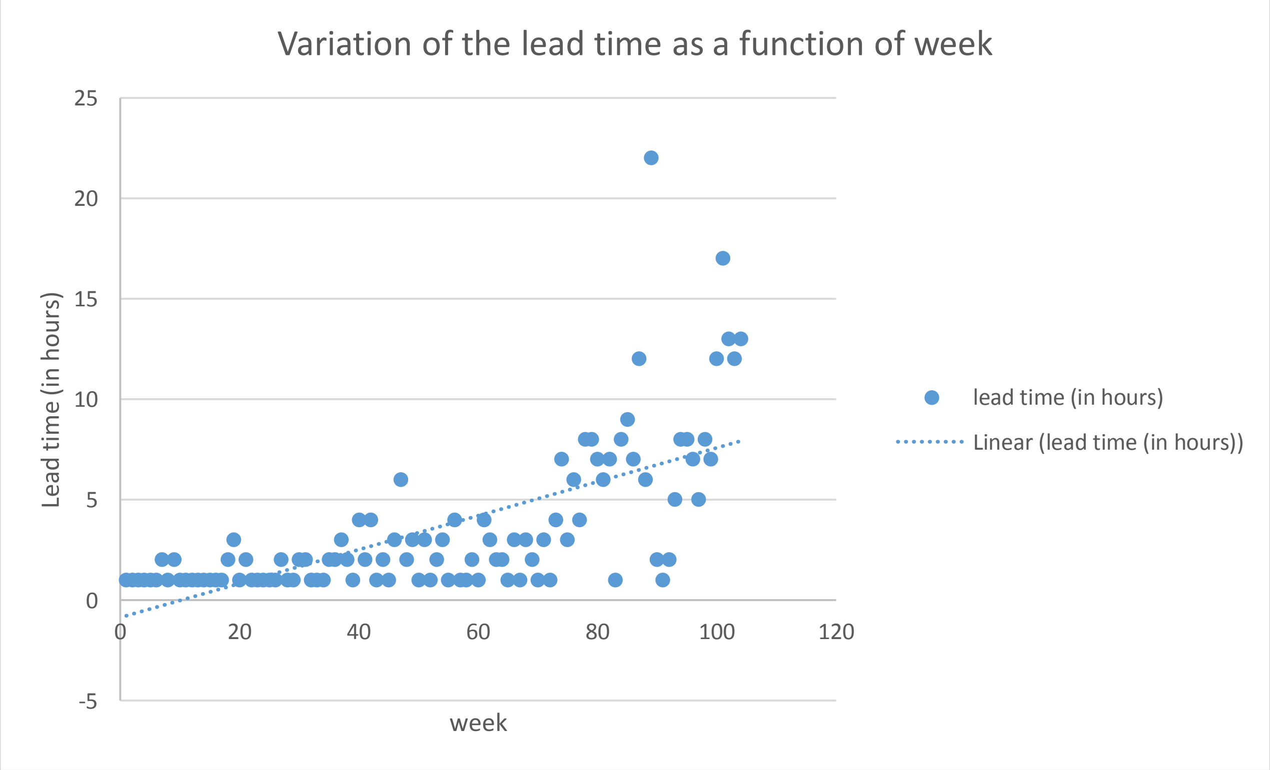
Appendix 3 (Squatter diagram for Demand)





Appendix 4 (Squatter diagram for lead time data)





Appendix 5 (Simulation output)

```
Variables in Main

a=200
b=0.0001661
B_bo_continuous=1.299
B_bo_hybrid=1.027
B_continuous=25.7
B_hybrid=20.36
B_periodic=25.7
c=0
d=2
dD_0dt=3.286
D_0=301
FW=0.0004808
f_c=0
F_correct=0.08
f_p=3.286
h=0.511
I_0=9.714
I_0_2=9.714
I_0_3=9.714
I_0_c=14.57
I_A_continuous=-120.8
I_A_periodic=-718.3
I_A_periodic_2=-3446
I_A_periodic_3=-7015
I_continuous=-113.9
I_hybrid=4765
I_periodic=-1173
I_periodic_2=1473
I_periodic_3=21514
j=201
k=125
LM=1
M_Lead_Time_bo_correct=
0.4234
n=8
N_=1.995
N_bo=5
N_continuous=3.579
N_hybrid=4.579
N_periodic=1
N_periodic_2=0.03571
N_periodic_3=0.008433
p=20
period=52
period_1=4
period_2=4
```

Production_Time_bo_correct=0.4234

Q_c=1.618

Q_p=301

Q_p_2=8429

Q_p_3=35698

r=8.323

R1=1

R1_0=2

R2=28

R3=118.6

S_s=-417.2

S_s_2=4983

S_s_3=28683

t=52

TC_continuous=2184

TC_hybrid=2445

TC_periodic=1948

TC_periodic_2=3907

TC_periodic_3=7331

T_b_continuous=0.8468

T_b_hybrid=0.8468

T_correct=0.005769

T_Lead_Time_bo_correct=37.08

T_Lead_Time_bo_correct_2=64.08

T_Lead_Time_bo_correct_3=154.7

T_no=0.01442

T_operation=0.1442

T_operation_bo=0.01442

T_su=0.01442

U=1.796

z_0=0

Appendix 6 (models' verification)

Note that the actual inventory cost (in Rand) of implementing both review policies should be multiplied by 1000.

Demand variation $\frac{\partial D}{\partial t}$	**Policy**	**Re-order point r**	**Order up to level (S)**	**Quantity ordered Q**	**Review – period R**	**Total cost per year**
3.286	*Periodic (R,S)*			9300	33	**R1867**
3.286	*Continuous (r,Q)*	12	2.4			**R1651**
	Difference					**R216**

Table 1 Simulation output (model verification) 1

Demand variation $\frac{\partial D}{\partial t}$	**Policy**	**Re-order point r**	**Order up to level (S)**	**Quantity ordered Q**	**Review – period R**	**Total cost per year**
300	*Periodic (R,S)*			8429	28	9807
300	*Continuous (r,Q)*	8.323		1.618		8684
	Difference					1123

Table 2 Simulation output (model verification) 2

Appendix 7 (Engineering Equation Solver program)

$$TC_{continuous} = k \cdot N_{continuous} + h \cdot \frac{1}{period} \cdot \int_{LM}^{period} (I_{continuous}) \, d t + j \cdot \frac{1}{period} \cdot \int_{LM}^{period} (B_{continuous}) \, d t$$

$$D_0 = a + \int_{LM}^{period} (b + f_c + f_p) \, d t$$

$$dD_{0dt} = b + f_c + f_p$$

$$B_{continuous} = \int_{LM}^{period} (B_{bo,continuous} \cdot T_{b,continuous}) \, d t$$

$$B_{bo,continuous} = \frac{1}{period} \cdot \int_{LM}^{period} \left[\frac{N_{bo}}{N_{continuous}} \right] d t$$

$$T_{b,continuous} = M_{Lead,Time,bo,correct} + Production_{Time,bo,correct}$$

$$M_{Lead,Time,bo,correct} = n \cdot \left[T_{su} + \left| Q_c \right| \cdot (T_{operation,bo} + F_{correct} \cdot T_{correct}) + T_{no} \right]$$

$$Production_{Time,bo,correct} = n \cdot (T_{su} + Q_c \cdot (T_{operation,bo} + F_{correct} \cdot T_{correct}) + T_{no})$$

$$I_{A,continuous} = I_0 + \frac{1}{N_{continuous}} \cdot \int_{LM}^{period} (I_{0,c} - dD_{0dt} \cdot t) \, d t$$

$$I_{continuous} = I_{A,continuous} + r - dD_{0dt} \cdot M_{Lead,Time,bo,correct}$$

$$r = I_0 - dD_{0dt} \cdot M_{Lead,Time,bo,correct}$$

$$N_{continuous} = \frac{D_0}{Q_c \cdot period}$$

$$Q_c = \int_{LM}^{period} \left[\frac{Q_c^{\ 2}}{k \cdot period \cdot D_0} \cdot (h \cdot I_{continuous} + j \cdot B_{continuous}) + \frac{Q_c}{D_0} \cdot dD_{0dt} + z_0 \right] dt + 1$$

$$TC_{periodic} = k \cdot N_{periodic} + h \cdot \frac{1}{period} \cdot \int_{LM}^{period} (I_{periodic}) \, dt + j \cdot \frac{1}{period} \cdot \int_{LM}^{period} (B_{periodic}) \, dt$$

$$TC_{periodic,2} = k \cdot N_{periodic,2} + h \cdot \frac{1}{period} \cdot \int_{LM}^{period} (I_{periodic,2}) \, dt + j \cdot \frac{1}{period} \cdot \int_{LM}^{period} (B_{periodic}) \, dt$$

$$TC_{periodic,3} = k \cdot N_{periodic,3} + h \cdot \frac{1}{period} \cdot \int_{LM}^{period} (I_{periodic,3}) \, dt + j \cdot \frac{1}{period} \cdot \int_{LM}^{period} (B_{periodic}) \, dt$$

$$B_{periodic} = B_{continuous}$$

$$N_{periodic} = \frac{D_0}{Q_p}$$

$$Q_p = D_0 \cdot R1$$

$$N_{periodic,2} = \frac{D_0}{Q_{p,2}}$$

$$Q_{p,2} = D_0 \cdot R2$$

$$N_{periodic,3} = \frac{D_0}{Q_{p,3}}$$

$$Q_{p,3} = D_0 \cdot R3$$

$$I_{A,periodic} = \int_{LM}^{period} (I_0 - dD_{0dt} \cdot T_{Lead,Time,bo,correct}) \; d\,t$$

$$I_{periodic} = I_{A,periodic} + S_s - T_{Lead,Time,bo,correct}$$

$$T_{Lead,Time,bo,correct} = R1 + n \cdot (T_{su} + Q_p \cdot (T_{operation,bo} + F_{correct} \cdot T_{correct}) + T_{no})$$

$$S_s = Q_p + I_{A,periodic}$$

$$I_{A,periodic,2} = \int_{LM}^{period} (I_{0,2} - dD_{0dt} \cdot T_{Lead,Time,bo,correct,2}) \; d\,t$$

$$I_{periodic,2} = I_{A,periodic,2} + S_{s,2} - T_{Lead,Time,bo,correct,2}$$

$$T_{Lead,Time,bo,correct,2} = R2 + n \cdot (T_{su} + Q_p \cdot (T_{operation,bo} + F_{correct} \cdot T_{correct}) + T_{no})$$

$$S_{s,2} = Q_{p,2} + I_{A,periodic,2}$$

$$I_{A,periodic,3} = \int_{LM}^{period} (I_{0,3} - dD_{0dt} \cdot T_{Lead,Time,bo,correct,3}) \; d\,t$$

$$I_{periodic,3} = I_{A,periodic,3} + S_{s,3} - T_{Lead,Time,bo,correct,3}$$

$$T_{Lead,Time,bo,correct,3} = R3 + n \cdot (T_{su} + Q_p \cdot (T_{operation,bo} + F_{correct} \cdot T_{correct}) + T_{no})$$

$$S_{s,3} = Q_{p,3} + I_{A,periodic,3}$$

$$R1 = 1$$

$$R2 = 28$$

$$R3 = R1_{0} + \frac{1}{N_{periodic}} \cdot \int_{LM}^{period} \left[\frac{R1^{2}}{period \cdot k} \cdot (h \cdot I_{periodic} + j \cdot B_{periodic}) + R1_{0} \right] dt$$

$$TC_{hybrid} = k \cdot N_{hybrid} + h \cdot \frac{1}{period} \cdot \int_{LM}^{period} (I_{hybrid})\, dt + j \cdot \frac{1}{period} \cdot \int_{LM}^{period} (B_{hybrid})\, dt$$

$$N_{hybrid} = N_{periodic} + N_{continuous}$$

$$N_{continuous} \cdot \frac{Q_{c}}{dD_{0dt}} + N_{periodic} \cdot \frac{Q_{p}}{dD_{0dt}} = 52 \cdot U$$

$$U = 0.9 \cdot N$$

$$I_{hybrid} = -I_{periodic} + Q_{c}$$

$$B_{hybrid} = \int_{LM}^{period} (B_{bo,hybrid} \cdot T_{b,hybrid})\, dt$$

$$B_{bo,hybrid} = \frac{1}{period} \cdot \int_{LM}^{period} \left[\frac{N_{bo}}{N_{hybrid}} \right] dt$$

$$T_{b,continuous} = T_{b,hybrid}$$

$$f_{c} = c \cdot t$$

$$f_{p} = d \cdot (1 - \mathbf{sin}(p \cdot t))$$

$$k = 125$$

$$h = 0.511$$

$$j = 201$$

$\text{period} = 52$

$\text{period}_1 = 4$

$\text{period}_2 = 4$

$a = 200$

$b = \frac{0.05}{D_0}$

$c = -0$

$d = 2$

$p = 20$

$z_0 = 10 \cdot 0$

$R1_0 = 2$

$n = 8$

$T_{\text{operation}} = 300 \cdot FW$

$T_{\text{operation,bo}} = 30 \cdot FW$

$F_{\text{correct}} = 0.08$

$T_{\text{correct}} = 12 \cdot FW$

$T_{\text{su}} = FW \cdot 30$

$T_{\text{no}} = FW \cdot 30$

$N_{\text{bo}} = 5$

$$FW = \frac{1}{5 \cdot 8 \cdot 52}$$

$$LM = 1$$

$$I_{0,c} = \frac{a - dD_{0dt} \cdot period}{2}$$

$$I_0 = \frac{a - dD_{0dt} \cdot t}{3}$$

$$I_{0,2} = I_0$$

$$I_{0,3} = I_0$$